Journal of Public Economics 45 (1991) 383–389. North-Holland

Immiserizing growth with semi-public goods under consistent conjectures

Robert M. Costrell*

University of Massachusetts, Amherst, MA 01003, USA

Received September 1989, revised version received December 1990

Population growth allows society to afford more public goods and, in this respect, improves per capita welfare possibilities. However, this note shows that with private provision of public goods, under consistent conjectures, population growth also exacerbates free-riding behavior. This ultimately reduces public good provision, when it should be increased, and population growth proves to be immiserizing. This striking result constrasts with the Nash model, where the improved welfare possibilities are at least partially exploited.

1. Introduction

Population growth allows society to afford more public goods and, in this respect, improves per capita welfare possibilities. However, this note shows that with private provision of public goods, under consistent conjectures, population growth also exacerbates free-riding behavior. This ultimately reduces public good provision, when it should be increased, and population growth proves to be immiserizing. This striking result contrasts with the Nash model, where the improved welfare possibilities are at least partially exploited.

Prior to establishing this result, we must consider the model of consistent conjectural equilibrium, for given population size. The model was first set out by Cornes and Sandler (1984) for the case of pure public goods. Sugden (1985) questioned the existence of an interior solution in this model. Cornes and Sandler (1985) responded by suggesting that a generalization to semi-public goods would ensure existence. Section 2 below verifies this conjecture for the simplest class of semi-public goods.

Once existence is established, the welfare analysis can proceed as outlined in section 3. The previous literature suggested that if an interior equilibrium existed under consistent conjectures, it would be inferior to the Nash

*I wish to acknowledge the helpful comments of Dale Ballou and of the particularly careful referees of the Journal.

equilibrium since underprovision of the public good would be more severe. That is because under consistent conjectures an individual believes his contribution to the public good will be at least partially offset by his neighbors' response. This result is immediately extended to the simple class of semi-public goods under consideration. Finally, population growth is considered, and the immiserization result obtains, provided the public nature of the good is relatively strong.

2. The model and existence of equilibrium

There are n individuals, each of whom consumes a semi-public good, Q, and a private good, y. Individual i's consumption equivalent of the semi-public good is

$$Q_i = q_i + \beta Q_{-i}, \quad 0 < \beta < 1,$$

where q_i is individual i's purchase of the good, and Q_{-i} is that of everyone else. Thus, own and other purchases of the semi-public good are perfect substitutes, but with a marginal rate of substitution, $0 < \beta < 1$, between them. The limiting cases of $\beta \to 0$ and $\beta \to 1$ correspond to pure private goods and pure public goods, respectively.

Individual i maximizes

$$U[y_i, Q_i] = U[y_i, q_i + \beta Q_{-i}], \tag{1}$$

subject to the constraint

$$y_i + q_i = I, \tag{2}$$

where income is I, and units have been chosen such that the prices of both goods are fixed at unity by supply conditions (under the usual assumption of a linear production frontier). Maximization is conditional on the conjectured response of the other consumers,

$$\lambda_i \equiv \mathrm{d}Q_{-i}/\mathrm{d}q_i, \tag{3}$$

so the slope of the perceived budget frontier in $y_i - Q_i$ space is $-(1 + \beta\lambda_i)$.[1]

Let U be twice continuously differentiable and strictly quasiconcave. Also, we assume that both goods y_i and Q_i are essential, in the sense that the MRS

[1]We follow the consistent conjectures literature here in requiring only that the consumer be correct about the value of this derivative at equilibrium (see below). In particular, the consumer is not so sophisticated as to treat this conjecture as variable (at least in the vicinity of equilibrium). That would render the budget frontier non-linear, possibly bending the wrong way.

approaches zero and infinity as we approach the axes.[2] Thus, individual i will choose an interior solution, provided the budget frontier is not flat or vertical $(0<(1+\beta\lambda_i)<\infty)$:

$$(1+\beta\lambda_i)U_Q[I-q_i, q_i+\beta Q_{-i}]=U_y[I-q_i, q_i+\beta Q_{-i}], \tag{4}$$

where subscripts on U denote partial derivatives. For use below, we also asume both goods are normal.

In forming the conjecture λ_i, individual i estimates how each of the other $(n-1)$ identical individuals will respond to an exogenous change in q_i. To do so, individual i differentiates j's equilibrium condition, while recognizing the impact on the $(n-2)$ other individuals, k. That is, individual i differentiates

$$\begin{aligned}&(1+\beta\lambda_j)U_Q[I-q_j, q_j+\beta\{q_i+(n-2)q_k\}]\\ &\quad =U_y[I-q_j, q_j+\beta\{q_i+(n-2)q_k\}]\end{aligned} \tag{5}$$

with respect to q_i, q_j, and q_k (given λ_j and n), and sets $\mathrm{d}q_j=\mathrm{d}q_k$ to capture the interactions among the $(n-1)$ identical other individuals. The resulting expression for the conjecture $\mathrm{d}q_j/\mathrm{d}q_i$ is used to construct $\lambda_i=(n-1)\,\mathrm{d}q_j/\mathrm{d}q_i$.

In symmetric equilibrium (if it exists) we have

$$(1+\beta\lambda)=\frac{[(1+\beta\lambda)U_{yQ}-U_{yy}]+[U_{yQ}-(1+\beta\lambda)U_{QQ}](1-\beta)[1+\beta(n-1)]}{[(1+\beta\lambda)U_{yQ}-U_{yy}]+[U_{yQ}-(1+\beta\lambda)U_{QQ}][1-\beta+\beta(n-1)]}, \tag{6}$$

where the partials are evaluated at values of y and Q that satisfy

$$(1+\beta\lambda)=U_y[y,Q]/U_Q[y,Q], \tag{7}$$

$$Q=(I-y)[1+\beta(n-1)]. \tag{8}$$

That is, the system (6)–(8) describes symmetric consistent conjectural equilibrium for λ, y, and Q, given β and n.[3]

We must now demonstrate the existence of an interior symmetric equilibrium, as suggested by Cornes and Sandler (1985), for semi-public goods. To do so, first consider (7) and (8) as a subsystem in y and Q, conditional on λ

[2]For example, the CES case qualifies, for any value of the elasticity of substitution. Actually, a weaker restriction will do as we approach the vertical axis, since it will be shown that the budget line will never be steeper than unity.

[3]For the limiting case of $\beta\to 1$ (pure public goods), this system is equivalent to the model given in Cornes and Sandler (1984), Sugden (1985) and Scafuri (1988). There are, however, computational errors in Cornes and Sandler's eq. (12) and Scafuri's eq. (5).

(as well as on β and n). The restrictions on preferences are sufficient to ensure an interior solution to this subsystem, provided $0<(1+\beta\lambda)<\infty$.[4] Denote such solution values as $y(\lambda;\beta,n)$ and $Q(\lambda;\beta,n)$.

Now consider (6). Following Sugden (1985), we re-express this in terms of the slope of the expansion path. That slope is

$$e(\lambda;\beta,n)\equiv[(1+\beta\lambda)U_{yQ}-U_{yy}]/[U_{yQ}-(1+\beta\lambda)U_{QQ}]>0, \tag{9}$$

by the assumption of normal goods, and where the partials are evaluated at $y(\lambda;\beta,n)$ and $Q(\lambda;\beta,n)$. Then (6) can be written as

$$(1+\beta\lambda)=\frac{e(\lambda;\beta,n)+(1-\beta)[1+\beta(n-1)]}{e(\lambda;\beta,n)+[1-\beta+\beta(n-1)]}. \tag{6'}$$

We will now show that (6′) is satisfied by some $\lambda\in(-1,0)$. As $\lambda\to-1$, the left-hand side approaches $(1-\beta)$, which is less than the right-hand side. As $\lambda\to 0$, the left-hand side approaches unity, which exceeds the right-hand side. Therefore, by continuity, there is a solution $\lambda\in(-1,0)$. Moreover, for such a solution, we have $0<(1-\beta)<(1+\beta\lambda)<1<\infty$, so the solution for y and Q is interior, as maintained.[5]

Finally, note that this generalizes Sugden's (1985) pure public good result that the consistent conjecture λ is negative (if both goods are normal). That is to say, consumers should recognize that an increase in their contributions to the semi-public good will be at least partially offset by a reduction from others.

3. Suboptimality and immiserizing growth

In this section we begin by comparing optimal, Nash, and consistent conjectural equilibrium configurations for any given population size. We then

[4]Consider $y\in(0,I)$. As $y\to 0$, $Q\to I[1+\beta(n-1)]>0$, and the right-hand side of (7) becomes infinite, so it exceeds the left-hand side. As $y\to I$, $Q\to 0$, and the right-hand side of (7) vanishes, so it falls below the left-hand side. Therefore, by continuity, an interior solution for y and Q exists to the subsystem (7) and (8), for strictly positive, finite values of $(1+\beta\lambda)$.

[5]For the pure public good case, $\beta=1$, existence of an interior solution is more problematic, as the previous literature discussed. In terms of the present analysis, the reason is that as $\lambda\to-1$, the left-hand side of (6′) vanishes (i.e. the budget line becomes flat), and the right-hand side also vanishes, since $e\to 0$, as the expansion path approaches the horizontal axis. To resolve interior existence, we need to know which side vanishes more rapidly, i.e. we need to compare the derivatives with respect to λ.

Sugden (1985) claimed that an interior solution will not exist 'under most reasonable assumptions' but only showed it for the Cobb–Douglas case (for reasonably-sized n). His non-existence result generalizes to the CES case, provided the elasticity of substitution is greater than or equal to unity (again, for reasonably-sized n), but it can be shown that an interior solution will exist (regardless of n) if the elasticity of substitution is less than unity. Thus, Sugden's claim was too broad.

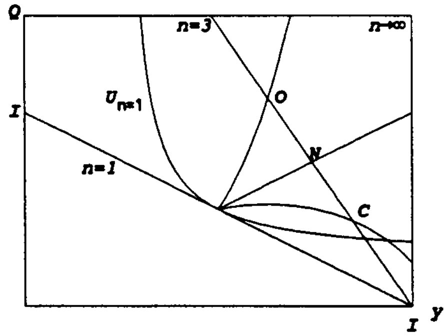

Fig. 1

consider population growth to establish the note's main result of immiserizing growth under consistent conjectures if the public nature of the good is relatively strong.

For a given population size, any symmetric configuration will lie on the economy's per capita transformation frontier between y and Q, given by (8). The optimal, Nash, and consistent conjectures configurations lie at different points on that frontier (see fig. 1, points O, N, and C along the frontier for $n=3$).

The optimal configuration, O, is the point of tangency between the transformation frontier and the indifference curve:

$$U_y[y,Q]/U_Q[y,Q]=[1+\beta(n-1)]. \tag{10}$$

The Nash and consistent conjectural configurations, N and C, are at tangencies with perceived budget frontiers of slope -1 and $-(1+\beta\lambda)$:

$$U_y[y,Q]/U_Q[y,Q]=1, \tag{11}$$

$$U_y[y,Q]/U_Q[y,Q]=(1+\beta\lambda), \tag{12}$$

where (12) reproduces (7). But since $\lambda<0$,

$$[1+\beta(n-1)]>1>(1+\beta\lambda),$$

for $n>1$, $\beta>0$, so the Nash configuration is southeast of the optimum, and the consistent conjectures configuration is southeast of Nash, as shown in fig. 1.[6] Thus, as we move from the optimum to Nash to consistent conjectures, we reallocate away from the semi-public good, and we move to less preferred

[6]The assumptions of strict quasiconcavity and normal goods imply that the MRS declines monotonically as we move down any negatively sloped (or vertical) line.

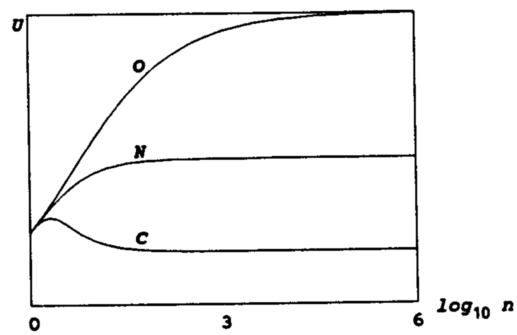

Fig. 2

positions. That is, the underprovision result is more pronounced for consistent conjectures than for Nash, as the previous literature suggested.

Now consider growth in population n, assuming that per capita income, I, is constant.[7] If we begin at $n=1$, the three configurations coincide at an interior point on the transformation frontier. As n grows, the frontier pivots on the y-axis and the three paths diverge.

The Nash path, which we already know to be the intermediate path, is simply a conventional expansion path, since the perceived budget line retains its slope of 1. Under the assumption of normal goods, it is positively sloped, and as $n \to \infty$ it approaches the right-hand side of the box in fig. 1. The optimal path must lie northwest of it, and can bend backwards. Welfare obviously improves monotonically along both the Nash and optimal paths.

The consistent conjectural path lies southeast of the Nash path, so as $n \to \infty$ it approaches the right-hand side of the box in fig. 1 at a point below the terminus of the Nash path. More precisely, note from (6′) that as $n \to \infty$, $(1+\beta\lambda) \to (1-\beta)$ (i.e. $\lambda \to -1$). Thus, the terminus of the consistent conjectural path is the point on the right-hand side of the box in fig. 1 where the MRS is $(1-\beta)$. This point lies below the terminus of the Nash path, where the MRS is unity, since the MRS declines monotonically as we move down the right-hand side of the box.[8]

The higher is β, the farther down the box is the terminus of the consistent conjectural path. For some $\beta \in (0,1)$, this terminus will fall below the indifference curve corresponding to $n=1$.[9] Therefore, if the public nature of the good is strong enough, the representative individual will choose a point

[7]In other words, assume the per capita production frontier between y and q is constant, as distinct from the per capita transformation frontier between y and Q, which is obviously not constant as n grows.

[8]Note that for both the Nash and consistent conjectural equilibria, $q \to 0$ as $n \to \infty$, but $n \to \infty$ faster, as Q does not vanish for $\beta < 1$.

[9]The terminus of the path will come arbitrarily close to the point $(I, 0)$ (where the MRS approaches zero) for β arbitrarily close to unity (i.e. pure public good).

inferior to that chosen when $n=1$. In other words, it is better to have no neighbors at all than to have so many that one cannot resist free-riding. Figs. 1 and 2 illustrate for a case of CES utility and $\beta=0.95$.[10]

4. Conclusion

It is said that no man is an island. Our immiserization result concludes that is a pity. If each man were an island, he would provide some positive quantity of the public good, since there would be no one else upon whom to free ride. As the population grows, however, the temptation to free ride grows, and rationally so. Immiserization follows if the public good is pure enough. This is quite unlike the Nash model, where at least some of the possibilities for improvement are exploited.

[10]The case illustrated has elasticity of substitution equal to $\frac{1}{2}$, and assigns equal weights to y and Q. For somewhat smaller βs, immiserization occurs relative to the optimal n, but not relative to $n=1$. For smaller βs yet (0.6 or below in this example), immiserization does not occur.

References

Cornes, Richard and Todd Sandler, 1984, The theory of public goods: Non-Nash behavior, Journal of Public Economics 23, 367–379.

Cornes, Richard and Todd Sandler, 1985, On the consistency of conjectures with public goods, Journal of Public Economics 27, 125–129.

Scafuri, Allen J., 1988, On consistency of conjectures in the private provision of public goods, Journal of Public Economics 37, 395–398.

Sugden, Robert, 1985, Consistent conjectures and voluntary contributions to public goods: Why the conventional theory does not work, Journal of Public Economics 27, 117–124.