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An Economic Analysis of College Admission Standards

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ABSTRACT *The effects of open or relaxed college admission standards vary across students. Among other effects, a relaxed standard is presumed to raise the number of college graduates, but it reduces the productivity of non-graduates. Since the most egalitarian standard-setters should accord least weight to the former and most weight to the latter, they should be least inclined to favor open admission. Moreover, the effect on the graduation rate is ambiguous, since some infra-marginal college attendees will become less likely to graduate. Finally, a lower admission standard reduces performance among students exceeding the graduation standard, by impairing their preparation for college work.*

Introduction

The perennially simmering debate over college admission standards has flared anew over the last decade of the education reform movement. The highly influential 1983 report, *A Nation at Risk*, recommended specific steps for 4-year institutions to raise their admission requirements. Bishop (1991a) reports increased selectivity from 1980 to 1985, a period of recovering Scholastic Aptitude Test (SAT) scores. However, as SAT scores drifted down again after 1985, and remediation grew at 4-year institutions, there have been renewed calls for raising admission standards, met by renewed resistance. Controversy over the City University of New York's decades-old policy of open admissions is the most publicized, but other universities and legislatures are also grappling with the fall-out from *de facto* open admissions.

The usual argument for open admission to college, or, more generally, lower admission standards, is that it will increase the number of college graduates, especially among those from disadvantaged backgrounds. Therefore, it is argued, open admission will have salutory effects on both aggregate productivity and the income distribution. It is recognized that many, perhaps most, of those students admitted under weaker requirements will not graduate, especially if graduation

R. M. Costrell, Department of Economics, Thompson Tower, University of Massachusetts, Amherst, MA 01003, USA. This paper extends the analysis of Costrell (1994), which restricts its attention to high-school graduation standards, but which examines many issues which are ignored here, such as the standard-setter's valuation of student leisure, decentralization of standard-setting, etc. An earlier version of this paper was presented at the 1993 meeting of the American Economic Association.

standards are maintained. However, the costs to these students are seen as a risk they should be allowed to bear in return for the possibility of success.[1]

There are several arguments against open admission. First, college graduation standards are not exogenous and there will be pressures to relax them. This will result in lower productivity of graduates, including many of those who would have enrolled and graduated under stricter admission and graduation requirements.

Second, lower admission standards reduce the incentive of high-school students to take demanding courses and perform well in them, thus reducing the productivity of high-school graduates. This argument applies most directly to the large number of students who enroll in college, but do not graduate. The productivity of this pool of students is governed by the admission standard, rather than the high-school or college graduation standards. In addition, if high-school graduation requirements are not exogenous, they may drop as well, reducing the productivity of the non-college bound.

Third, if lower admission standards reduce the preparation of students who were already college bound and if college graduation standards are maintained, then these students will find graduation more difficult. Some of those who would have been admitted under stricter standards may now find themselves unable or unwilling to do what is necessary to graduate. Thus, the college graduation rate may not rise much, and it might even fall.

Finally, some of those who are now less well prepared for college work may still choose to graduate, exceeding the graduation standard, but by a lesser amount.

This paper presents a theoretical framework for weighing these costs and benefits of lower admission standards, and also a micro-economic analysis of these incentive effects. The first key element to the analysis is the assumed structure of information flows utilized by employers. For the non-college-bound and college non-graduates, information is limited to the high-school diploma or college admission. Educational achievement, as indicated by employment tests or grades, are not examined by employers in this analysis, perhaps out of fear of Equal Employment Opportunity Commission (EEOC) action, as argued by Bishop (1988, 1990a,b, 1991a,b). Information flows for college graduates may also be limited, but the model does not require it.

Second, it is assumed that students maximize utility from future income and current leisure. Together with the assumption of limited information flows, this implies that non-college bound students have no incentive to do anything more than meet the high-school graduation requirement. Also, college enrollees will have no labor market incentive to exceed the admission requirement, though they may choose to do so as part of an inter-temporal allocation plan for study effort, discussed below.

Third, in evaluating the optimality of admission standards, it is assumed that social welfare depends only on student future income, and not on student leisure. Open admission is optimal if the corner solution it represents maximizes social welfare.

To fix ideas, it may be helpful to consider the case where a large state university dominates the state's higher education. Community colleges are omitted from the analysis, as is the array of more selective private institutions. College graduation standards are set by one entity (e.g. the faculty), but the admission standard is set by a separate entity—the admissions office or, perhaps, the state legislature. High-school graduation standards are set by another state-wide entity or the localities.

The next section begins with a framework for analyzing optimal admission policy. The following two sections then provide a micro-economic analysis of student behavior, to examine the effect of the admission standard on the college graduation rate; the first of these two sections is based on optimization under certainty, while the second considers student uncertainty regarding the difficulty of completing college, along the lines of Manski's (1989) model. The final section analyzes the affect of the admission standard on the productivity of college graduates. The main results are:

(1) If a marginally lower admission standard leads to lower college or high-school graduation standards, this only reduces social welfare if the graduation standards are too low to begin with. This will be the case, for example, if standard-setters are more egalitarian than society at large.

Holding fixed the productivity of the non-college-bound and of college graduates:

(2) Open admission is less likely to be optimal if the goals are egalitarian than if they are non-egalitarian. This result, which reverses the usual presumption, is driven by the fact that egalitarian goals put less weight on the number of college graduates than on the income of non-graduates.
(3) A sufficient set of conditions for open admission to be suboptimal is found to be plausible. These are: (i) the college graduation standard is no higher than its optimum; (ii) the number of college students who would just meet the graduation standard, under open admission, is no greater than the number of college non-graduates; and (iii) college graduation is more sensitive to the graduation standard than to the admission standard.
(4) Under student certainty, a drop in admission standards will increase the number of college graduates, as proponents claim, but for a new reason: more college students who would have settled for non-graduation find that with lower admission standards, non-graduation is insufficiently remunerative.
(5) Under uncertainty regarding the difficulty of college work, there are three effects of a lower admission standard on the number of graduates: (i) more college enrollees, of whom some will graduate; (ii) the improved incentive to graduate, discussed above, among some students who would attend college anyway; and (iii) by contrast, a reduced incentive to graduate among others who would attend college, but who now find excessive the extra work required for graduation.
(6) Among students who continue to exceed the graduation requirement, lower admission standards will also lead some to be less well prepared, resulting in reduced performance, due to imperfect inter-temporal substitutability of leisure.

Concluding remarks consider the impact of improved information flows to employers.

A Framework for Welfare Analysis of Admission Standards

The three standards under consideration are the high-school graduation standard, w^h, the college admission standard, w^a, and the college graduation standard, w^c, all defined as the productivity level required to earn the educational credential. Beginning with the most general formulation, social welfare can be expressed as a reduced form function of these standards: $V\ (w^h, w^a, w^c)$. The effect of a marginal change in w^a, then, is

$$\frac{dV}{dw^a} = \frac{\partial V}{\partial w^a} + \frac{\partial V}{\partial w^h}\frac{dw^h}{dw^a} + \frac{\partial V}{\partial w^c}\frac{dw^c}{dw^a} \tag{1}$$

Endogenous Graduation Standards

Most of this paper is devoted to the direct effect of a marginal change in the admission standard, $\partial V/\partial w^a$, but first it briefly considers the indirect effect through endogenous graduation standards, the last two terms of (1). Writing down this expression reveals the obvious point, which has, to my knowledge, gone hitherto unremarked, that if graduation standards are at the optimum, ($\partial V/\partial w^h = \partial V/\partial w^c = 0$), then it makes no difference if or how they respond to a marginal change in the admission standard. This is simply an application of the envelope theorem.[2]

Thus, the argument that a relaxed admission standard reduces welfare by driving down graduation standards (dw^h/dw^a, $dw^c/dw^a > 0$) rests on the assumption that graduation standards are too low to begin with ($\partial V/\partial w^h$, $\partial V/\partial w^c > 0$). If so, then the last two terms of (1) will be positive and the argument will be valid.

There may be good reasons to believe that graduation standards are indeed too low. I have argued elsewhere (Costrell, 1994) that excessively egalitarian standard-setters and decentralization may be forces for suboptimal high-school graduation standards.

Nor is college immune from egalitarian impulses or competitive pressures. Most recently, post-modernist trends in some humanities departments have rendered standards themselves unfashionable. Other departments may then feel driven by competition for students to also reduce standards.[3] The ironic lesson of the envelope theorem here is that faculty complaints of laxness in the admissions office, pressuring them to dilute graduation standards, are only relevant when their own house is already out of order. Finger-pointing aside, however, it remains the case that endogenous graduation standards may well be an important avenue by which relaxed admission standards reduce welfare.

Exogenous Graduation Standards

For the remainder of this paper, graduation standards will be taken as exogenous, so the focus is on $\partial V/\partial w^a$. To analyze this more closely, four levels of educational attainment are considered: high school non-graduate, high school graduate, college non-graduate and college graduate. As stated above, it is assumed that information flows are such that students have no incentive to exceed the standard for any given level of attainment, with the possible exception of college graduates.[4] Thus, high-school non-graduates earn the minimal productivity level, w^0 (which is exogenous and assumed identical for all students), high-school graduates earn w^h, college non-graduates earn[5] w^a and college graduates earn $w \geq w^c$.

The proportion of students who reach productivity levels w^i, and no higher, are denoted n^i, $i = 0,h,a,c$; and if students have an incentive to exceed w^c, they are distributed with density $f(w)$, such that $n^+ \equiv \int_{w>w^c} f(w)dw = 1 - n^0 - n^h - n^a - n^c$. The proportions n^i and density $f(w)$ depend on the standards set, as students weigh rewards and required effort, to determine their level of attainment.

Social welfare is assumed to be additively separable, aggregating an increasing concave function, $h(w^j)$, of each individual's income. If h is strictly concave, then the social welfare function is egalitarian to some degree; if h is linear, then the goal is income maximization. Thus, we have

$$V(\omega) = h(w^{o})\, n^{o}(\omega) + h(w^{h})\, n^{h}(\omega) + h(w^{a})\, n^{a}(\omega) + h(w^{c})\, n^{c}(\omega) + \int_{w > w^{c}} f(w;\omega) h(w) \mathrm{d}w \tag{2}$$

where $\omega \equiv (w^{h}, w^{a}, w^{c})$. We wish to evaluate the conditions under which relaxed admission standards raise welfare, $\partial V/\partial w^{a} < 0$. The specific question of open admission's optimality (a corner solution) is addressed by evaluating $\partial V/\partial w^{a}$ as $w^{a} \to w^{h}$.[6]

Assume, as the specific models below imply, that the marginal high-school non-graduate's decision is affected by the high-school graduation standard, but not the standards for college admission or graduation, i.e. $n^{0} = n^{0}(w^{h})$. Assume also, for the moment, that students have no incentive to exceed w^{c}, or that those who do are unaffected by admission and graduation standards. Then we have,

$$\frac{\partial V}{\partial w^{a}} = [h(w^{c}) - h(w^{a})] \frac{\partial n^{c}}{\partial w^{a}} - [h(w^{a}) - h(w^{h})] \frac{\partial n^{h}}{\partial w^{a}} + h'(w^{a}) n^{a} \tag{3}$$

To interpret this expression, begin with the second term. If a lower admission standard induces more students to meet that standard, going beyond the high school diploma, then $\partial n^{h}/\partial w^{a} > 0$. Thus, the second term is negative for $w^{a} > w^{h}$, tending to raise social welfare as admission standards are lowered, since more students exceed the productivity level of high school graduates.

If, in addition, the number of college graduates rises as the admission standard falls, then $\partial n^{c}/\partial w^{a} < 0$ and the first term is negative, too. The case for open admission rests on this term, rather than the second term, since open admission means $w^{a} \to w^{h}$, and the second term vanishes.[7]

On the other hand, the third term is positive, as a lower admission standard reduces the productivity of those who would have attended college anyway, but who do not graduate. These are the high school students whose incentives are adversely affected, as argued by opponents of lower standards.

To evaluate the optimality of open admission, consider

$$\operatorname{sgn} \partial V/\partial w^{a}|_{w^{a} \to w^{h}} = \operatorname{sgn} (\partial n^{c}/\partial w^{a})/n^{a} + h'(w^{h})/[h(w^{c}) - h(w^{h})] \tag{4}$$

The interesting point here is that it is more likely positive for an egalitarian social welfare function than an income-maximizing one. For strictly concave h, the second term exceeds $1/(w^{c} - w^{h})$, which is the value under income maximization ($h(w) = w$). Thus, open admission is less likely to be optimal with egalitarian goals, since more weight is given to the adverse effect on the income of college non-graduates than to the presumably favorable effect on the number of graduates.

Of course, this is the reverse of the usual presumption that open admission advances egalitarian goals. That presumption appears to ignore the trade-off just described, and to rest instead on a trade-off between the number of graduates and their income. This latter trade-off would be generated by the endogenous effect on graduation standards, discussed above, or by diminished productivity of those who exceed the graduation standards, discussed below.

In general, then, the standard-cutter should weigh the presumably favorable effect on the number of graduates against the adverse effects on the incomes of both graduates and non-graduates. This means the relationship between the degree of egalitarianism and preference for open admission is not necessarily monotonic. However, it remains the case that the most egalitarian goals (approaching the Rawlsian limit) are advanced by tending to the interests of those with the lowest income—the college non-graduates here—and that means rejecting open admission.

Open admission is also less likely to raise welfare if the college graduation standard is low, since that reduces the benefit of graduating. This consideration allows us to find and interpret a set of sufficient conditions for open admission to be suboptimal. If we suppose the college graduation standard is no higher than the social optimum, then as $w^{a} \to w^{h}$

$$0 \leq \frac{\partial V}{\partial w^{c}} = h'(w^{c})n^{c} + [h(w^{c}) - h(w^{a})]\,\frac{\partial n^{c+}}{\partial w^{c}} \tag{5}$$

where $n^{c+} \equiv n^{c} + n^{+}$, the number of students who meet or exceed the graduation standard. Using (5) in (3) implies [8]

$$\operatorname{sgn}\partial V/\partial w^{w} \geq \operatorname{sgn} 1 - [h'(w^{c})/h'(w^{a})]\,[n^{c}/n^{a}]\,[(\partial n^{c+}/\partial w^{a})/(\partial n^{c+}/\partial w^{c})] \tag{6}$$

If each of the three bracketed terms on the right-hand side is below unity, then the expression is positive and open admission reduces social welfare. The first of these terms is less than unity for egalitarian social welfare, reproducing the point made earlier. The second term is the ratio of college students who choose to meet, but not exceed, the graduation requirements, relative to non-graduates. The ratio n^{c+}/n^{a} is currently about unity,[9] so n^{c}/n^{a} is lower, and would likely be lower yet under open admission. The third bracketed term is less than unity if college graduations are more sensitive to college graduation standards than to admission standards. For some or all of these three reasons, it is certainly plausible, and perhaps quite likely, that $\partial V/\partial w^{a} > 0$ as $w^{a} \to w^{h}$. If so, open admission reduces social welfare, provided the college graduation standard is no higher than its optimum.

Finally, consider the effect of the admission standard on the students exceeding w^{c}. Then, instead of (3), we have, from (2)

$$\frac{\partial V}{\partial w^{a}} = [h(w^{c}) - h(w^{a})]\,\frac{\partial n^{c}}{\partial w^{a}} - [h(w^{a}) - h(w^{h})]\,\frac{\partial n^{h}}{\partial w^{a}} + h'(w^{a})n^{a} + \int_{w > w^{c}} [h(w) - h(w^{a})]\,\frac{\partial f(w;\omega)}{\partial w^{a}}\,dw \tag{3'}$$

The new term here is the last one. It represents the effect of admission standards on those who exceed the graduation standard, among those who previously did not and also among those who previously did. The first and last terms together give the effects on the number of college graduates, n^{c+} (the subject of the next two sections), and their incomes, too (the subject of the last section).

A Model of the Graduation Rate under Certainty

To flesh out the foregoing analysis, student response to the admission standard is now explicitly modelled, particularly $\partial n^{c+}/\partial w^{a}$. As the welfare analysis suggested, an important but insufficient condition for the optimality of open admission is $\partial n^{c+}/\partial w^a < 0$. This section shows it to hold under certainty (with a caveat given below), but it need not hold once student uncertainty regarding the difficulty of college work is introduced, in the next section.

Student utility rests on future income, high-school educational labor (L_1) and college educational labor (L_2), where educational labor is the time and effort devoted to meeting standards. Preferences are well behaved and differ by student. L_1 and L_2 are perfect substitutes in the educational production functions, which also vary by student, reflecting differences in complementary inputs (e.g. home inputs, ability). Notation is simplified by expressing student j's educational production function in inverse form, i.e. the labor required to meet a given level of proficiency, $L_1 + L_2 = l^j(w)$.[10] Assume that l^j is convex.

Perfect Substitutability in Preferences

Consider the limiting case of perfect substitutability between high-school and college leisure, so student j's preferences are $U^j(w, L_1 + L_2)$. Students are faced with the choice of five outcomes:

$U^j(w^0,$	$l^j(w^0) = 0)$	High-school non-graduation
$U^j(w^h,$	$l^j(w^h))$	High-school graduation
$U^j(w^a,$	$l^j(w^a))$	College non-graduation
$U^j(w^c,$	$l^j(w^c))$	College graduation at the standard
$U^j(w > w^c,$	$l^j(w))$	College graduation exceeding the standard

Figure 1 shows the configuration of students on the margin between one level of educational attainment and the next, who are indifferent between the two. These are the students who will respond to marginal changes in standards, and they will only respond to standards of adjacent educational levels. For example, the number of students just meeting the college graduation standard, n^c, will respond to w^c and w^a, but not to w^h, while the number of students exceeding the standard, n^+, will respond to w^c, but not to w^a or w^h.

Moreover, it is readily shown that students on the margin between two levels of educational attainment will rise to the higher level with a drop in the standard of either the lower level or the higher level. For example, consider students on the margin between just graduating from college and not graduating:

$$U^j(w^a, l^j(w^a)) = U^j(w^c, l^j(w^c)) \tag{7}$$

Depicted in Figure 1 with indifference curve ac, these students find $dU^j(w^c, l^j(w^c))/dw^c < 0$, so a drop in w^c leads them to prefer graduating: $\partial n^{c+}/\partial w^c = -\partial n^a/\partial w^c < 0$. They also find $dU^j(w^a, l^j(w^a))/dw^a > 0$, so a drop in w^a leads them to graduation, too: $\partial n^{c+}/\partial w^a = \partial n^c/\partial w^a < 0$, as proponents of open admissions claim.

It should be noted, however, that the logic here behind $\partial n^c/\partial w^a < 0$ is rather different from the usual one. The positive effect of lower admission standards on college graduations in this model does not come from new entrants (though the number of college entrants does rise, $\partial(n^a + n^{c+})/\partial w^a = -\partial n^h/\partial w^a < 0$). Instead, the increased graduations come from students who would enter college anyway, but now find it less rewarding to stop short of graduation: the lower pay, w^a, outweighs the lower work requirement, $l^j(w^a)$.

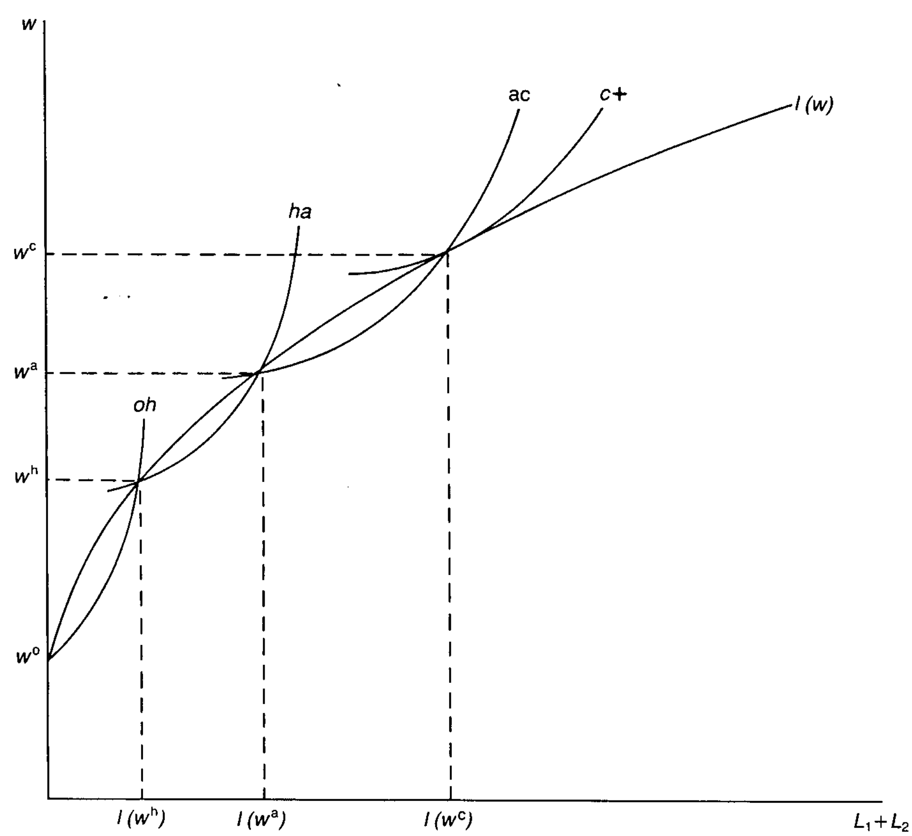

Figure 1. Certainty, perfect substitutability.

To summarize, the pattern of partials in this model is:

$$\underset{+\ \ 0\ \ 0}{n^o(w^h,w^a,w^c)},\ \underset{?\ \ +\ \ 0}{n^h(w^h,w^a,w^c)},\ \underset{-\ \ ?\ \ +}{n^a(w^h,w^a,w^c)},\ \underset{0\ \ -\ \ ?}{n^c(w^h,w^a,w^c)},\ \underset{0\ \ 0\ \ -}{n^+(w^h,w^a,w^c)}$$

Imperfect Substitutability in Preferences

With imperfect substitutability, the analysis of marginal college graduates is modified by the fact that the utility of college graduation may now depend on the admission requirement. That was not the case under perfect substitutability, where a marginal graduate (who will not exceed w^c) finds $U^j(w^c, L_1 + L_2 = l^j(w^c))$ to be independent of w^a.

Under imperfect substitutability, the utility of graduating is $\max_{w,L_1} U^j(w \geq w^c, L_1 \geq l^j(w^a), L_2 = l^j(w) - L_1)$. Students who choose to exceed the admission requirement (e.g. by taking Advanced Placement courses in high school) are still unaffected by changes in the requirement. However, among students for whom the requirement is binding, a drop in it renders college graduation more attractive. This reinforces the conclusion above that $\partial n^{c+}/\partial w^a < 0$, as the proponents of open admissions argue.

In particular, imperfect substitutability seems to underlie the usual scenario proffered in favor of open admission: the student from a disadvantaged background, with fewer complementary inputs to the educational production function, who is nonetheless motivated to do the extra work required to become a college graduate. Under perfect substitutability, such a student is indifferent between doing the work in high school or college, so the admission standard is irrelevant. Under imperfect substitutability, however, a high admission standard may force the student to frontload more of the work than is desirable or feasible, so college attendance becomes unattractive.

Imperfect substitutability, however, also raises another possibility, less favorable to open admission. In some cases, high schools do not offer students the opportunity of exceeding admission requirements. Advanced Placement courses, for example, are not always available, and sometimes schools will even try to eliminate such courses on the grounds that they are 'elitist'. In such cases, the admission requirement $L_1 \geq l^j(w^a)$ is supplemented by the school's restriction $L_1 \leq l^j(w^a)$, so the student who chooses to attend college has no option other than $L_1 = l^j(w^a)$.

If the admission standard is very low, so the school's restriction binds, too much of the work required to graduate from college is backloaded on to the college years: college graduation becomes less attractive as the admission standard is lowered. Examples can be constructed where this results in $\partial n^{c+}/\partial w^a > 0$, which would undercut the case for lower admission standards.[11] The interpretation suggested here is that raising admission standards can force some high schools to offer students the opportunity to prepare themselves sufficiently for college work. This argument is analogous to the one sketched earlier regarding endogenous high-school graduation standards, w^h: a rise in admission standards may not only induce them to raise the floor, w^h, but also to raise the ceiling, w^a, if one exists. This, in turn, can result in more college graduates.

The Graduation Rate under Uncertainty

The certainty formulation offers some insights, but it has at least one obvious drawback: all decisions are made in high school. In particular, the decision of some to attend college, but not graduate, is made in advance: no college drop-outs are disappointed.

In this section, Manski's (1989) model of college attendance under student uncertainty regarding the work entailed in graduating from college is adopted and applied to the problem of admission standards. In so doing, the usual argument for a lower admission standard is formalized: more students will attend college and some of them will graduate. This argument was missing in the certainty formulation, since none of the marginal college attendees graduate.

Also, as before, some infra-marginal college attendees will be more inclined to graduate, as lower admission standards render non-graduation less remunerative and thereby less attractive. However, unlike the certainty model, there will now be other infra-marginal college attendees who reach the opposite conclusion: they are more attracted by the reduced labor for non-graduation and they become less likely to graduate.

The point of the uncertainty model can be made in the simple case of perfect substitutability. Following Manski, students receive a signal, once in college, about how much work is required to graduate, and then act accordingly. To get that information, they must first meet the admission requirement, in high school.

The decision process works backwards. First, one ascertains the conditions under which one would graduate, given that one has met the admission requirement. Then one calculates the expected utility over all possible college outcomes. This is compared with the alternative, which is to stop with the high-school diploma.

As before, the labor requirements to meet the high-school graduation standard or the college admission standard are certain: individual j needs $L_1 \geq l^j(w^h)$ or $L_1 \geq l^j(w^a)$ respectively. The labor required to produce a college graduate level of productivity, however, is uncertain prior to enrolling in college:

$$L_1 + L_2 = l^j(w) - \varepsilon, \qquad w \geq w^c \tag{8}$$

Let ε be distributed with mean zero, so $l^j(w)$ is interpreted as the expected labor requirement. A high draw for ε means the student can get by with less work.[12]

First, consider the student's options, should he/she choose to graduate. Let

$$w^{j\star}(\varepsilon) \equiv argmax U^j(w, l^j(w) - \varepsilon), \qquad w \geq w^c \tag{9}$$

represent the optimal point on the student's *ex post* production function at or above w^c. Then, working backwards, define $\varepsilon^{j\star}$ as the threshold ε above which the student will choose to graduate, rather than drop out:

$$U^j(w^a, L_1) \equiv U^j(w^{j\star}(\varepsilon^{j\star}), l^j(w^{j\star}(\varepsilon^{j\star})) - \varepsilon^{j\star})$$

Expected utility from attending college is:

$$E(U^j) = \Phi^j(\varepsilon^{j\star})\, U^j(w^a, L_1) + \int_{\varepsilon > \varepsilon^{j\star}} \phi^j(\varepsilon)\, U^j(w^{j\star}(\varepsilon), l^j(w^{j\star}(\varepsilon)) - \varepsilon)d\varepsilon$$

where ε's density and distribution functions are ϕ^j and Φ^j.

One can immediately establish that prospective college students will choose not to exceed the admission standard, $L_1 = l^j(w^a)$, since $E(U^j)$ decreases in L_1. There is no point for the student to exceed the admission requirement, since, under the assumption of perfect substitutability,[13] he/she is indifferent between frontloading the work in high school or backloading it in college, in the event he/she decides to graduate. If the student draws a low ε and does not graduate, any excess L_1 in high school would have been wasted, since w^a is exogenous to the student. Thus, we can replace L_1 in the expressions above:

$$U^j(w^a, l^j(w^a)) \equiv U^j(w^{j\star}(\varepsilon^{j\star}), l^j(w^{j\star}(\varepsilon^{j\star})) - \varepsilon^{j\star}) \tag{10}$$

$$E(U^j) = \Phi^j(\varepsilon^{j\star})\, U^j(w^a, l^j(w^a)) + \int_{\varepsilon > \varepsilon^{j\star}} \phi^j(\varepsilon)\, U^j(w^{j\star}(\varepsilon), l^j(w^{j\star}(\varepsilon)) - \varepsilon)d\varepsilon \tag{11}$$

The impact of a drop in w^a on three groups of students is now considered: those students who are just on the margin between attending college and not; those students who are above the margin, but not by much; and those students who are well above the margin.

Students who are on the margin are characterized by

$$U^j(w^h, l^j(w^h)) = E(U^j) \tag{12}$$

These students will be induced to rise above the margin by a drop in w^a if $dE(U^j)/dw^a < 0$. It is readily shown that this is the case. To see this, first note, from (11), that sgn $dE(U^j)/dw^a =$ sgn $dU^j(w^a, l^j(w^a))/dw^a$ (since terms in $d\varepsilon^{j\star}/dw^a$ cancel). Then note that $E(U^j)$ is a weighted average of $U^j(w^a, l^j(w^a))$ and the higher utilities, $U^j(w^{j\star}(\varepsilon), l^j(w^{j\star}(\varepsilon)) - \varepsilon)$, for $\varepsilon > \varepsilon^{j\star}$. For marginal students, satisfying (12), it then follows that $U^j(w^h, l^j(w^h)) > U^j(w^a, l^j(w^a))$. This, in turn, implies

$dU^j(w^a, l^j(w^a))/dw^a < 0$ for these students. (In Figure 1, their indifference curve through $(l^j(w^a), w^a)$ is steeper than curve ha.)

Thus, the number of students entering college rises as the college admission requirement drops: $\partial(n^a + n^{c+})/\partial w^a = -\partial n^h/\partial w^a < 0$, as under certainty. The difference is that under certainty, none of these marginal enrollees graduates. Under uncertainty, some of them will graduate,[14] assuming $\phi^j(\varepsilon)$ is sufficiently diffuse. This supports the story of proponents of lower admission standards.

Now consider the infra-marginal students. A drop in w^a will not affect their decision to attend college, but will affect their probability of completing it. Equation (10) implies[15]

$$\operatorname{sgn} d\varepsilon^{j\star}/dw^a = \operatorname{sgn} dU^j(w^a, l^j(w^a))/dw^a \tag{13}$$

If a drop in the admission standard makes it more attractive to stop short of graduating ($dU^j(w^a, l^j(w^a))/dw^a < 0$), student j will be less likely to graduate: $\varepsilon^{j\star}$ will rise and the student will be less likely to draw $\varepsilon > \varepsilon^{j\star}$.

As we have seen, this will in fact be the case for those students who are at the margin of college attendance and, by extension, not far above it. These students are not inclined to graduate anyway, unless they get a rather good draw of ε. The trade-off of less income for additional leisure, by settling for non-graduation, becomes more attractive to them as the admission standard drops. This result, which is perhaps the most novel of this paper, means that a lower admission standard leads to fewer graduates among these near-marginal students.

There will also typically be a third group of students, quite a bit above the margin of attending college, whose most preferred point on the expected production function, $(w, l^j(w))$, is above $(w^a, l^j(w^a))$,[16] e.g. curve ac in Figure 1. For this group, $dU^j(w^a, l^j(w^a))/dw^a > 0$: non-graduation's trade-off of less income for additional leisure becomes less attractive as the admission standard drops. As in the certainty case above, these likely graduates will be even more likely to graduate as w^a drops.

To summarize, then, we cannot be sure of the crucial sign of $\partial n^{c+}/\partial w^a$. The model gives two reasons why it might be negative, as the proponents of lower standards assume. First, as they argue, a fall in w^a increases the number of students attending college, some of whom will graduate. Second, in a result that has not been recognized before, more of the most highly motivated students will find it worthwhile to graduate, rather than settle for a lower income, w^a. On the other hand, in another result that has not been formalized before, a drop in admission standards leaves those college students who are closer to the margin less inclined to do the extra work required to graduate. If this group is large enough, it is possible that $\partial n^{c+}/\partial w^a > 0$, undercutting the argument for lower admission standards.

Admission Standards and Graduate Productivity

Having considered the effect of admission standards on the number of graduates, we now turn to the effect on the performance of those who would have graduated anyway. Aside from endogenous graduation standards, discussed earlier, the issue arises only if the information flow to employers suffices to create an incentive for some to exceed the graduation standard. In terms of the welfare analysis of the first section, the subject is the last term of (3'). More precisely, it is that portion of the term which represents the behavior of those who change the level at which they graduate, rather than those who cross the line between non-graduation and graduation.

Under perfect substitutability, the optimal choice over $w \geq w^c$, given in (9), will not be affected by w^a. This includes the certainty case $\varepsilon \equiv 0$, depicted in Figure 1.

The crux of the matter, then, is imperfect substitutability. The point can be made in the certainty model. The question is the behavior of

$$w^{j\star}(w^a) \equiv \operatorname{argmax}\{\max_{L_1}[U^j(w \geq w^c, L_1 \geq l^j(w^a), L_2 = l^j(w) - L_1)]\} \tag{14}$$

among those whose solution to (14) is preferable to not graduating from college. Within this group, a marginal change in w^a will have no effect on $w^{j\star}$ among those whose optimal L_1 is unconstrained and also among those whose optimal w is strictly constrained. However, if there are students who are constrained by the admission standard, but not by the graduation standard, $w^{j\star}$ will depend on w^a. Their solution will be characterized by

$$\mathrm{MRS}^j_{L_2,w}(w, L_1 = l^j(w^a), L_2 = l^j(w) - l^j(w^a)) = l^{j\prime}(w) \tag{15}$$

Totally differentiating, we find

$$\frac{dw^\star}{dw^a} = \frac{l'(w^a)\,(\partial\mathrm{MRS}/\partial L_2 - \partial\mathrm{MRS}/\partial L_1)}{\partial\mathrm{MRS}/\partial w + l'(w)\,\partial\mathrm{MRS}/\partial L_2 - l''(w)} \tag{16}$$

where the notation for individual j has been suppressed. If w and collegiate leisure are normal goods, for given L_1, then $\partial\mathrm{MRS}/\partial w$, $\partial\mathrm{MRS}/\partial L_2 < 0$, so the denominator is negative. In the limiting case of perfect substitutability, $\partial\mathrm{MRS}/\partial L_1 = \partial\mathrm{MRS}/\partial L_2$, and the numerator vanishes, so $dw^\star/dw^a = 0$, as stated earlier. Short of perfect substitutability, however, $\mathrm{MRS}_{L_2,w}$ will be more sensitive to L_2 than to L_1, so the numerator will be negative. Thus, under imperfect substitutability, $dw^\star/dw^a > 0$.

That is, a drop in the admission standard leads to reduced performance among admission-constrained students who are willing to exceed the graduation standard. Allowing them to enter college with less preparation makes it harder for them to produce as much.[17] In (3'), a drop in w^a redistributes downward the density $f(w)$. This makes it more likely that $\partial V/\partial w^a > 0$, favoring a rise in the admission standard, rather than a relaxation.

Conclusion

This paper has identified the varying impact of a marginally relaxed admission standard on several groups of students. In roughly ascending order of student-preferred productivity levels:

(1) Productivity *falls* among non-college-bound students, if the high-school graduation standard is endogenously relaxed (second term of equation (1)).
(2) Productivity *rises* among previously non-college-bound students who now attend college, but do not graduate, provided the admission standard still exceeds the high-school graduation standard (second term of equation (3')).
(3) Productivity *rises* among previously non-college-bound students who now graduate from college (first and last terms of (3')).
(4) Productivity *falls* among college non-graduates (third term in (3')).
(5) Productivity *falls* among college students who were unlikely to graduate and

become even less likely to do so, due to the increased attractiveness of non-graduation (first and last terms of (3')).

(6) Productivity *rises* among college students who were likely to graduate and become even more likely to do so, due to the reduced attractiveness of non-graduation (first and last terms of (3')).

(7) Productivity *falls* among those who just meet the college graduation standard, if the graduation standard is endogenously relaxed (third term of equation (1)).

(8) Productivity *falls* among those who exceed the graduation standard, but are constrained by the admission standard, as they enter less well prepared (last term of equation (3')).

The analysis is driven by imperfect information flows from students to employers, student uncertainty regarding the difficulty of college work and imperfect inter-temporal substitutability of leisure. Little can be done about the latter two, but it may be worth considering the implications of improving information to employers. Bishop, for example, advocates such measures for the non-college-bound as providing plastic-encoated high-school transcripts to students, for presentation to employers; modifying the legal barriers to employment testing and to employer examination of high-school transcripts; and granting distinct high-school diplomas, like the New York Regents' diploma versus the general diploma.

Consider the simple case of perfect inter-temporal substitutability of leisure, under certainty. Depicted in Figure 1, improved information flows would fill in the gaps between w^0, w^h, w^a and w^c. All students, then, would simply choose their tangency. Neither the college graduation standard nor the admission standard would have any significance. To be sure, this full information solution is not a social optimum. It leads to the students' first-best optimum, but society typically does not accord student leisure the same weight as students do (the social welfare function used in this paper assumes zero weight is given to student leisure). There is an agency problem here, and incomplete information flows, coupled with high standards, can, in some cases, be a strategy for minimizing student leisure. However, Costrell (1994) has shown that in the relatively simple case of the high-school graduation standard, full information does typically raise social welfare, given a high degree of student heterogeneity.

Even with perfect information flows to employers, there remains the ineradicable uncertainty of high-school students regarding the difficulty of college work. In such a world, the graduation standard w^c becomes meaningless, but the admission standard w^a still affects behavior. It forces students to precommit to a certain level of productivity, if they are to attend college and possibly obtain good news on how well they can perform. Admission-constrained students would rather not precommit to such a high level of productivity in the event that they would choose, *ex post*, to do no work in college, either due to a bad draw of ε or to a very good draw.[18] The standard can be manipulated as the price for obtaining information.

Raising the admission standard still has conflicting effects. On the negative side, it reduces the number of college attendees. These admission-constrained students will now drop down to a lower productivity level than the admission standard. Some of these would have had good draws in college and become quite productive. On the positive side, those infra-marginal attendees who are nonetheless admission constrained will be forced into a higher fall-back productivity, in the event of draws that would lead them to do no additional work.

Finally, as we have seen, even if there is certainty regarding the difficulty of college work and perfect information flows to employers, the admission standard still affects behavior due to imperfect inter-temporal substitutability of leisure. It does so by forcing students to inter-temporally reallocate their study effort. That is, even though the college graduation standard is meaningless here, the admission standard $L_1 \geq l^j(w^a)$ is still the price attendees must pay for the option of choosing $L_2 > 0$.

The standard, then, retains conflicting effects. On the negative side, a higher standard decreases the number of college attendees (leaving aside possible high-school ceilings on student effort, discussed earlier). This reduces productivity among marginal students by constraining them to choose $L_2 = 0$. On the positive side, all admission-constrained infra-marginal attendees will be induced to choose a higher level of productivity. In the previous section, this was only true of those who were also unconstrained by the graduation standard; here, under perfect information, no one is constrained by the graduation standard. That is, forcing admission-constrained students to be better prepared for college work will lead them to greater productivity once they are through, all the more so when they are free to choose any level of college productivity.

In short, the case for higher admission standards rests on exploiting the positive incentive effects that derive from incomplete information flows from students to employers, student uncertainty regarding the difficulty of college work, and imperfect inter-temporal substitutability of leisure. In an imperfect world, where students value their leisure more than society at large does, admissions officers should weigh these gains against the costs of barring some motivated students from immediate access to 4-year colleges and universities.

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Notes

1. Some of the costs are borne by others, since higher education is subsidized, but these costs will not be considered here.
2. In the case of a discrete drop in the admission standard, such as a jump to open admission, it is also insufficient simply to argue that this reduces welfare by driving down graduation standards. It might well be that the optimal graduation standards drop discretely, too, in which case the question is whether graduation standards are driven down too far, not whether they are driven down at all.
3. Sabot and Wakeman-Linn (1991) find that grade inflation in humanities departments attracts students away from the less inflated science departments.
4. This exception is motivated by the greater attention paid to transcripts (or at least majors) by employers of college graduates than of high-school graduates.
5. It is assumed that college non-graduates do not spend enough time or effort in college to improve their productivity at all.
6. If social welfare is raised by lowering w^a to w^h, this raises the question (not pursued here) of why a high-school diploma is required for college admission at all, i.e. why restrict $w^a \geq w^h$?
7. This result clearly rests on the strong assumption mentioned in note 5, that the productivity of college drop-outs is unaffected by their stay in college. This consideration mitigates the results in (4) and (6), but not the rest of the paper.
8. Under stated assumptions, $\partial n^{c+}/\partial w^c = \partial n^c/\partial w^c - f(w^c)$ and $\partial n^{c+}/\partial w^a = \partial n^c/\partial w^a$.
9. This is excluding associates' degrees; it is much lower than unity including them in n^a.

10. The welfare analysis above restricts $l^j(w^0) = 0$ for all j.
11. In this case, it is still true that the number of college entrants rises as the admission standard drops $(\partial(n^a + n^{c+})/\partial w^a = -\partial n^h/\partial w^a < 0)$, but fewer of them graduate.
12. Two potential difficulties with the formulation in (8) call for comment. First, a negative ε would seem to require the student to exert additional effort, $L_2 > 0$, just to regain the productivity level which was already attained in high school, a rather non-sensical feature. However, the range of (8) is restricted to $w \geq w^c$, so this difficulty only arises for students who exceed the college graduation requirement while in high school, $L_1 \geq l^j(w^c)$. Moreover, it will be shown below that it is irrational for students to exceed the admission requirement prior to college, so $L_1 = l^j(w^a) < l^j(w^c)$ and the problem does not arise.

    The opposite difficulty occurs when ε is so strongly positive that (8) implies a student can graduate without doing any work in college, and the non-negativity constraint $L_2 \geq 0$ binds. This rather unlikely complication can be readily modelled, but the cost in ease of exposition and notation seems excessive. Instead, it is simply assumed that the graduation requirement sufficiently exceeds the admission requirement that $l^j(w^c) - l^j(w^a) > \max(\varepsilon)$.
13. The fact that $E(U^j)$ is strictly decreasing in L_1 here suggests that the result would survive some relaxation of perfect substitutability.
14. It is readily established that among those students at or near the margin, for whom $dU^j(w^a, l^j(w^a))/dw^a < 0$, $\varepsilon^{j\star} > 0$, so if the distribution of ε is symmetric around 0, less than half of these students will graduate. Those students who are well above the margin, for whom $(w^c, l^j(w^c))$ is preferred to $(w^a, l^j(w^a))$, will have $\varepsilon^{j\star} < 0$ and more than half of them will graduate.
15. When totally differentiating the right-hand-side of (10), note that $dU^j/dw^j = 0$ at the indicated point, if $w^{j\star}(\varepsilon^{j\star}) > w^c$, an interior solution to (9); and $dw^{j\star}/d\varepsilon = 0$, if $w^{j\star}(\varepsilon^{j\star}) = w^c$, a boundary solution. Either way, the result in (13) obtains.
16. For these students, the decision to attend college is easy, since even the worst case scenario of not graduating, w^a, is preferred to stopping with the high school diploma, w^h.
17. Those admission-constrained graduates who are unwilling to exceed the graduation standard will simply reallocate their leisure from college to high school, without reducing their performance.
18. The simplifying assumptions made in note 12, where there exists a meaningful college graduation standard, w^c, do not apply under perfect employer information.

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