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ABSTRACT: It is well-known that public pension plans exhibit substantial cross-subsidies, both within cohorts, e.g. from early leavers to those who retire at the “sweet spot” ([Costrell and Podgursky, 2010](#)), and across cohorts, through unfunded liabilities. However, the cross-subsidies within and across cohorts have never been provided in an integrated format. This paper provides such a framework, based on the gaps between normal cost rates for individuals (building on [Costrell and McGee, 2017](#)) and the uniform contribution rates for the cohort. Since the unfunded liabilities and associated cross-subsidies across cohorts derive from overly optimistic actuarial assumptions, we focus on the historically most important such assumption, the rate of return. We present two main findings. First, an overly optimistic assumed return understates the degree of redistribution within the cohort. Second (building on [Costrell, 2016b](#)), persisting with an overly optimistic assumed return leads to steady-state contribution rates that exceed the true normal cost (let alone the low-balled rate), i.e. cross-subsidies from the current cohort to past cohorts. Using the case of California, we show how that negative cross-subsidy can easily swamp all positive cross-subsidies within the cohort, as contributions exceed the value of benefits received by even the most favored individuals – those who retire at the “sweet spot.”

KEYWORDS: teacher pensions

JEL CODE: I22, H75

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CROSS-SUBSIDIZATION OF TEACHER PENSION COSTS: THE IMPACT OF ASSUMED MARKET RETURNS

I. INTRODUCTION AND SUMMARY

The funding plans for traditional teacher pension systems are built upon a highly uneven set of benefits, varying widely in value by age of entry and exit. As we have shown elsewhere ([Costrell and McGee, 2017](#)), this variation can be informatively represented by the individual normal cost rates to fund these benefits. For example, early leavers may earn benefits worth 5 percent of salary per year while the benefits of those who stay until retirement are worth 25 percent. The aggregate normal cost rate – say 15 percent – is applied to all, uniformly, generating cross-subsidies of 10 percent of pay from the 5-percenters to the 25-percenters. If the funding plan pans out – i.e., the actuarial assumptions are fulfilled – these cross-subsidies, by design will sum to zero. In short, some of the contributions by or for the early leavers help pay the benefits of the career teachers.

However, as has been painfully evident for some time, the actuarial assumptions have often been overly optimistic – most notably the assumed return on investment – generating unfunded liabilities ([Munnell, Aubry, and Cafarelli, 2015](#); [Costrell, 2016a](#)). Specifically, this means that the normal cost rates have been low-balled, failing to cover currently accruing liabilities, and pushing the costs to future cohorts. This has two redistributive effects: (1) overstating the assumed return and understating the normal costs understates the cross-subsidies *within* cohorts, since (as we will show) the assumed return has a greater effect on long-termers than short-termers; and (2) low-balling the normal cost rate generates a cross-subsidy *between* cohorts, through the unfunded liability, as we have been observing with greater and greater force ([Backes, et. al., 2016](#)). Moreover, as [Costrell \(2016b\)](#) has shown, if the plans persist with overly optimistic assumptions, these cross-subsidies to past cohorts will persist in steady state. That is

because the faulty assumptions leave the system underfunded in steady state, so the shortfall in investment returns must be offset by contributions that exceed the true normal cost. Indeed, as we will show, with the example of California teachers (CalSTRS), it can easily be the case that steady state contributions exceed the value of benefits for even the most favored individuals, those who receive the greatest cross-subsidy within their cohort: *all* individuals in the current cohort may be losers, providing net negative cross-subsidies to past cohorts.

The plan of the paper is as follows. First, we will review the math of individual normal costs and the associated cross-subsidies within cohorts embedded in the funding plan, which is to say, when assumptions are fulfilled. We illustrate by the case of CalSTRS under recent, overly optimistic investment assumptions (as acknowledged by CalSTRS), and then under lower assumed returns. This will allow us to examine the impact on the cross-subsidies *within* cohorts, if the plan were to move from high assumed returns, low-balling normal cost, to lower assumed returns, with contributions equal to the “true” normal cost, which is much higher. Next, we will consider the impact of failed investment assumptions, where the plan persists with inflated assumed returns. Initially, this pushes costs onto future cohorts, but as the future arrives, the contributions not only rise toward and beyond the true normal cost, they actually stay that high in steady state. We estimate the steady-state contribution rate, including the amortization on unfunded liabilities, using the math developed in [Costrell \(2016b\)](#), together with data from CalSTRS valuation reports. This allows us to present an integrated picture of steady-state cross-subsidies within and between cohorts. The result can easily be quite striking, with net negative cross-subsidies for all members of the current cohort.

II. INDIVIDUAL NORMAL COST RATES AND CROSS-SUBSIDIZATION

Pension plans calculate the normal cost rate at the aggregate level, to fund a cohort's benefits as they accrue. Embedded within the calculation, however, are individual cost rates, based on age of entry and exit ([Costrell and McGee \(2017\), Appendix](#)), but they are not publicly reported. Specifically, consider an individual of type (e,s) , where e is the age of entry and s (for separation) is the age of exit. For each type (e,s) , we identify an individual normal cost rate, n_{es} , as a constant percent of salary over one's career. We calculate this rate to generate a stream of contributions sufficient to fund the individual's future benefits. That is, the PV of contributions must equal the PV of benefits.

Formally, for an individual of type (e,s) , we must have $n_{es}W_{es} = B_{es}$, where W_{es} is the PV of earnings (so $n_{es}W_{es}$ is the PV of contributions) and B_{es} is the PV of benefits (both evaluated at entry). It immediately follows that the individual cost rate is the ratio of the PV of benefits to that of earnings:

$$(1) \quad n_{es} = B_{es}/W_{es}.$$

This is the rate that, applied to the individual's annual earnings over her career, would prefund her benefits. It represents the value of her benefits earned annually, as a percent of earnings – an *individual fringe benefit rate* for pensions.

If we compare individuals with different entry and exit ages, (e,s) , we find their cost rates, n_{es} , vary widely. In general, for any given e , n_{es} rises with s , from the point of vesting up through a peak value retirement age. This is a manifestation of the well-known back-loading of benefits that favors long-termers under traditional pension formulas based on final average

salary, FAS ([Costrell and Podgursky, 2009, 2010](#)). The variation in n_{es} with e , for any given s , is less obvious, and can go either way.¹

In general, traditional FAS plans levy a joint (employee plus employer) contribution rate, c , that is uniform, independent of the individual's normal cost. Thus, we effectively have a system of cross-subsidies ($n_{es} - c$), positive or negative, as the value of individual benefits exceeds or falls short of the contributions made by or for her. The four steps we present, to understand the system of cross subsidies when optimistic assumptions are not fulfilled take us through the analysis of how the true and assumed investment return affect both n_{es} and c . First we present the array of individual cost rates and the aggregate normal cost under the optimistic assumed return r' , call them n'_{es} and n' . These generate an array of cross-subsidies ($n'_{es} - n'$) within cohorts, embedded within the optimistic funding plan. Next we consider a funding plan without rose-colored glasses, with a lower assumed return r^* – we shall refer to it as the “true” return – generating higher normal cost rates for the individuals and the aggregate, call them n^*_{es} and n^* . This will allow us to assess how the funding plan's system of *within*-cohort cross-subsidies is distorted by the optimistic discount rate, ($n'_{es} - n'$) vs. ($n^*_{es} - n^*$). Third, considering n^*_{es} as the true cost of individual benefits, we consider the system of cross-subsidies, within and between cohorts, while contributions are held at the low-balled aggregate normal cost, ($n^*_{es} - n'$). Finally, we consider the steady-state contribution rate c'^* , formed under the optimistic assumption, r' when the true rate is r^* , where c'^* includes the amortization payments on the steady-state unfunded liabilities. Thus, the steady-state system of cross-subsidies is ($n^*_{es} - c'^*$).

¹ Later entrants with the same exit age have shorter service, so their pension and its PV, B_{es} , is lower, but so is that of their earnings, W_{es} . Thus, the pattern can go either way, over different ranges of s , and different discount rates.

III. CROSS-SUBSIDIES WITHIN COHORTS, EMBEDDED IN THE FUNDING PLAN

Consider an entering cohort, whose entrants vary by age at entry, e , and projected age of exit, s .² Denote the joint frequency of e and s , among entrants, as p_{es} . Let us now consider a *uniform* normal cost rate, call it n , applied to all members of the cohort (of varying entry ages) throughout their careers (of varying length). It can readily be shown that the uniform cost rate required to fund the cohort's projected benefits is:

$$(2) \quad n = \sum_e \sum_s n_{es} (p_{es} W_{es}) / (\sum_e \sum_s p_{es} W_{es}).$$

This is the ratio of the PV of the cohort's benefits³ to the PV of the cohort's earnings: the same relationship we saw for the individual normal cost rate holds for the cohort as a whole. This expression also shows, importantly, that n is a weighted average of individual normal cost rates n_{es} across ages of entry and exit. The weights for n_{es} are $(p_{es} W_{es}) / (\sum_e \sum_s p_{es} W_{es})$, representing the share of type (e, s) in the cohort's PV of earnings.⁴

The deviations $(n_{es} - n)$ are positive and negative, corresponding to whether the cost of funding any individual's benefit exceeds or falls short of the cohort's uniform contribution rate, n . They constitute cross-subsidies. Moreover, by the nature of averages, these cross-subsidies must add up to zero, when properly weighted, by shares of the cohort's PV of earnings:

$$(3) \quad \sum_e \sum_s (n_{es} - n) (p_{es} W_{es}) / (\sum_e \sum_s p_{es} W_{es}) = 0.$$

Specifically, this means that the funding plan, by definition, embeds cross-subsidies *within* cohorts only – not *between* the present cohort and past or future cohorts.

² The results in this section can be shown to apply, not simply to a single entering cohort, but to any cohort, past or present, or the full set of such cohorts working their way over time through the workforce, under a given benefit formula and set of actuarial assumptions ([Costrell and McGee \(2017\)](#)).

³ Substituting $n_{es} = B_{es}/W_{es}$ into the numerator gives $\sum_e \sum_s p_{es} B_{es}$.

⁴ These are not the exact weights used in actuarial practice, but are consistent with the approach (see [Costrell and McGee \(2017\), Appendix](#)).

Illustration with the CalSTRS Plan

We illustrate with the California State Teacher Retirement System (CalSTRS) plan. We estimate the individual normal cost rates, $n_{es} = B_{es}/W_{es}$, for all entry and exit ages, $e, s = 20, \dots, 75$. We base our calculations on the CalSTRS actuarial assumptions (slightly modified, as explained below) and benefit formula. The actuarial assumptions cover wage growth (merit salary increases by entry age and years of service, and inflation), discount rate, exit rates for retirement (by age and years of service), exit rates prior to retirement (by years of service), and mortality rates (for female actives and future retirees). These assumptions are provided in the 2015 annual valuation report ([CalSTRS, 2016a](#)),⁵ supplemented with more granular data (annual rates vs. selected rates) from CalSTRS. We also use CalSTRS data on the age distribution of entrants. The benefit formula is delineated in the annual valuation report, as well as the member handbook ([CalSTRS, 2016b](#)). This includes the retirement eligibility conditions, age-specific multipliers (described below), cost of living adjustments (COLA), employee contribution rate, discount rate, and interest rate on refunds.

Since B_{es} and W_{es} are proportional to the entry wage, $n_{es} = B_{es}/W_{es}$ is independent of it, so we can normalize B_{es} and W_{es} per dollar of entry wage. For W_{es} we have:

$$(4) W_{es} = \sum_{a=e}^s (1+r)^{(e-a)} w_{a|e} = \sum_{a=e}^s (1+r)^{(e-a)} \prod_{\alpha=e}^a (1 + g_{e,(\alpha-e)}^w),$$

where $w_{a|e}$ is the wage at age a , given entry at age e (per dollar of entry wage), which is governed by the wage growth series $g_{e,(\alpha-e)}^w$ (merit plus inflation) by entry age and service, as given by CalSTRS actuarial assumptions, and r is the discount rate (discussed further below).

Benefits can be in the form of a pension or refund of employee contributions.⁶ If a

⁵ CalSTRS has since revised its assumptions, as discussed below.

⁶ We leave aside disability and death benefits, which comprise about 5 percent of normal cost, less than 1 point.

teacher takes the refund, she forgoes the pension (or possibility of future pension), and receives the cumulative value of the employee (but not employer) contributions, with accumulated interest at the rate set by CalSTRS. Teachers who leave before vesting, without the expectation of returning and becoming eligible for a pension, would certainly take the refund because it is the only benefit to which they are entitled. Teachers who leave after vesting, but too young to draw a pension, may either take the refund or leave the money in the fund to draw a pension in the future, upon reaching an eligible age. Finally, teachers who leave service and are eligible for an immediate pension, may still choose the refund, although it is generally not financially prudent to do so. If a teacher takes the refund, B_{es} is the present value of the cumulative employee contributions with interest i as determined by CalSTRS, discounted back to entry, $PV(Refund_{es})$:

$$(5) PV(Refund_{es}) = c^{ee} [\sum_{a=e}^s (1+i)^{(s-a)} w_{a|e}] / (1+r)^{(s-e)},$$

where c^{ee} is the employee contribution rate, (9.205 percent) and $i = 4.5$ percent in 2015.⁷

If a teacher takes the pension, B_{es} is the present value of the stream of pension payments, weighted by the survival probabilities, discounted to entry, $PV(Pension_{es})$. In general terms,

$$(6) PV(Pension_{es}) = \sum_{a>s} (1+r)^{(e-a)} b(a|e,s) f(a|s),$$

where $b(a|e,s)$ is the pension payment at age a , given entry and exit ages e and s , and $f(a|s)$ is the survival rate to age a , conditional on survival to exit age s . The payments $b(a|e,s)$ begin with a starting pension equal to an age-specific multiplier \times years of service $(s - e) \times$ FAS (average of final 3 years, normalized per dollar of entry wage), augmented annually with a 2.0 percent simple COLA. Specifically, we consider the “2% at 62” program for new hires (since 2013), with multipliers ranging from 1.16 percent at age 55 to 2.0 percent at age 62 and 2.4 percent at

⁷ For the 2016 valuation report, interest on member accounts is reduced to 3.00 percent.

65, after 5-year vesting. Vested employees who withdraw before age 55 but do not cash out must defer the pension to at least age 55, and we assume they collect then.⁸ Finally, we assume that teachers choose the refund or pension to maximize the present value of their benefits:⁹

$$(7) B_{es} = \text{Max}[PV(\text{Refund}_{es}), PV(\text{Pension}_{es})].$$

Together with W_{es} from (4), we have $n_{es} = B_{es}/W_{es}$. This gives us the contribution rate required, over one's career, to fund the benefits of an individual entering at age e and exiting at age s .

CalSTRS' assumed return (and discount rate) through 2015 was $r' = 7.5$ percent, and is in the process of being reduced.¹⁰ To illustrate the impact of a reduction in the assumed return, we evaluate n_{es} and the associated cross-subsidies at $r' = 7.5$ percent and $r^* = 6.0$ percent. The latter is a further cut than CalSTRS has yet undertaken, but is illustrative of the possible true return.

Variation in Normal Cost Rates by Age of Entry and Exit

We first consider the case under the optimistic assumed return of $r' = 7.5$ percent. Figure 1 depicts the normal cost rates, n'_{es} , for selected ages of entry (representative of all ages) and all exit ages. The variation is wide, from 6.9 percent to 21.1 percent (the full range, for entry ages not shown, is 6.4 to 24.4 percent¹¹). The pattern for any given entry age (e.g., age 25) is depicted *along* each curve, as the exit age varies. Prior to vesting, and for some years beyond, the benefit is the refund of employee contributions. The normal cost rate, therefore, starts at the employee contribution of 9.205 percent: each curve begins at the dashed horizontal line representing that rate. The cost rate then gently declines, falling slowly below the employee contribution rate.

⁸ CalSTRS assumes they defer to age 60. Our modification eliminates a discontinuity in the individual normal cost rate between age 54 and 55 that arises for lower discount rates.

⁹ CalSTRS assigns probabilities of taking the refund which may not maximize PV. Our modified assumption eliminates a precipitous drop in the individual normal cost rate upon vesting, due to suboptimal cash-outs.

¹⁰ The discount rate has been cut to 7.25 percent for the 2016 valuation and 7.00 percent thereafter.

¹¹ The higher rates are for unusual entry ages, later than 45.

That is because the interest credit of 4.5 percent is below the fund's assumed return, 7.5 percent. The contribution rate needed to cover the refund falls as this difference accumulates.

At a certain point, the pension becomes more attractive than the refund. For example, a 25-year-old entrant reaches that point at age 47; at this age the pension would still be deferred until eligibility at 55, but exceeds in PV the value of the employee refunds. Beyond that point, the normal cost rate rises as the deferral becomes shorter, and then, beyond age 55, there is no deferral, but n'_{es} continues to rise as the age-specific multiplier grows. Each year of delayed retirement beyond 55 is a year of forgone pension payments, but prior to age 65, the growth in the multiplier outweighs this effect. After age 65 the multiplier stops growing, and the normal cost declines. This pattern is reflected in Figure 1 along each curve, corresponding to any given entry age. In addition to the variation *within* entry-age cohorts, Figure 1 also depicts the (vertical) variation *across* entry ages for the same exit age.

Cross-Subsidy Rates and the Degree of Redistribution

The wide variation among individual cost rates contrasts with the uniform contribution rate. That is the weighted average, n' , as given in (2) for $r = r'$. This is the normal cost rate that will fund the benefits of each or all cohorts, past and present, represented in the current workforce, under the current benefit formula and the optimistic assumed return. We calculate n' to be 13.8 percent of pay, depicted in Figure 1 as the solid horizontal line.¹² The deviations of individual cost rates from n' represent the cross-subsidy rates, $(n'_{es} - n')$. Those above the line receive cross-subsidies from those below the line. For example, the extreme points depicted for n'_{es} , of 6.9 and 21.2 percent, represent cross-subsidies of -6.9 and +7.3 percent of pay.

¹² This is about one percentage point below CalSTRS' 2015 calculation of the normal cost rate for new hires, after netting out death and disability benefits.

These cross-subsidies are built into the funding plan, under the optimistic assumed return. For those individuals below the solid line, the plan is counting on using some or all of the employer contributions – plus, for many, part of the employee contributions – along with all of the assumed returns to help finance the benefits of those above the line. As shown in (3), the weighted sum of the cross-subsidy rates, is zero.

Using the joint frequencies of entrants, p_{es} , and their shares of lifetime earnings, $(p_{es}W_{es})/(\sum_e \sum_s p_{es}W_{es})$, we can calculate a few summary statistics. Those who provide the cross-subsidies (those below the line in Figure 1) comprise 68 percent of entrants and account for 49 percent of their lifetime earnings, and those who receive the cross-subsidies are the remainder. How large are the cross-subsidies? Taken together, the losers provide cross-subsidies that total –2.6 percent of their lifetime earnings. That is the average cross-subsidy rate for those below the line in Figure 1, weighted by shares of lifetime earnings. The winners receive cross-subsidies that average +2.5 percent. One can readily verify the zero-sum result: $0.51 \times 2.5\% - 0.49 \times 2.6\% = 0.0\%$. Thus, in all, taking absolute values of the cross-subsidies, 2.6 percent of total income is redistributed ($0.51 \times 2.5\% + 0.49 \times 2.6\%$), about one-fifth of the total normal cost.

How do the Normal Cost Rates and Cross-Subsidies Vary With the Discount Rate?

The analysis above was based on $r' = 7.5$ percent. CalSTRS has already recognized that this was overly optimistic and has started to reduce r . Here we consider what the funding plan would look like if r were further reduced to $r^* = 6.0$ percent, which we take as illustrative of the “true” rate of return (it is comparable to recent 10-year rolling averages). Whether this is too high or too low, it will serve to show the direction of the impact of the discount rate. At this

point we are still considering the impact on the distribution of cross-subsidies embedded in the funding plan, which is to say that all the cross-subsidies are *within* cohorts.

The individual normal cost rates under $r^* = 6.0$ percent are depicted in Figure 2. As one would expect, all the normal cost rates are increased from those depicted in Figure 1. Since the funding plan is no longer counting on such large investment returns, the contributions must be higher to fund the benefits. This much is well-known. What is perhaps less widely understood is that a drop in the assumed return will increase the degree of redistribution embedded in the funding plan. Stated alternatively, an over-optimistic assumed return not only underfunds the plan, but also understates the true degree of redistribution, as we will show.

Specifically, we are interested in how the impact of r on the normal cost rates n_{es} varies, since that will determine the impact on the cross-subsidies and the degree of redistribution. If all n_{es} were to rise by the same amount with a drop in r , then n would rise by approximately the same amount;¹³ the cross-subsidies ($n_{es} - n$) would remain nearly unchanged, and so would the degree of redistribution. This is not the case. To understand the impact on redistribution, we examine more closely how n_{es} is affected for those who receive the benefits in different forms.

For those who take refunds of the employee contributions, reducing the plan's assumed return on those contributions toward the (lower) interest credited to the refunds raises the cost closer to the employee contribution rate, c^{ee} . This can be seen directly:

$$(8) \ n_{es}^{refund} = c^{ee} \left[\sum_{a=e}^s (1+i)^{(s-a)} w_{a|e} \right] / \left[\sum_{a=e}^s (1+r)^{(s-a)} w_{a|e} \right].$$

More to the point, we can see from inspection of (8) that for given e , the impact of r on n_{es} increases with s . The longer the period of employee contributions, the greater will be the impact

¹³ There would be some small effect from the change in weights attached to the individual cost rates, induced by the change in r .

of the assumed return on the accumulated difference between that return and any given interest rate. This can be seen comparing Figure 1 with Figure 2, where the assumed return drops toward the interest on the refunds, and the normal cost rate rises toward c^{ee} .

The next group we consider is those who would take the refund under a high discount rate, but switch to the pension under a low discount rate. For example, as mentioned above, 25-year-old entrants would take the refund up to age 47 under the high discount rate, but as Figure 2 shows, they would switch to the deferred pension as early as age 42 under the low discount rate, since the deferral is less costly in PV terms. Thus, for those in this age bracket, 42 – 47, the impact of a drop in r is even greater than its effect on the PV of refunds. That is, the impact continues to widen, at an accelerated pace, with longer service.

Finally, we consider those who would take the pension under either discount rate. Their normal cost rate is:

$$(9) n_{es}^{pension} = \sum_{a>s} (1+r)^{(e-a)} b(a|e,s) f(a|s) / \sum_{a=e}^s (1+r)^{(e-a)} w_{a|e}.$$

A lower discount raises both the numerator and the denominator, but the impact is greater on the numerator since the flows are farther out (after separation, rather than before). Hence a lower discount rate raises the normal cost rate for this group, too, as we would expect. More importantly for our inquiry, the expression also suggests that the impact of the discount rate increases with the exit age, s , so long as the normal cost rate itself increases with s , i.e. up to the point of maximum normal cost. For if extending the age of exit raises the numerator proportionately more than the denominator (so n_{es} rises with s), then the greater impact of r on the numerator than the denominator will result in a greater impact of r on n_{es} as s rises.

These results are illustrated in Figure 3, which depicts n_{es} for 25-year-old entrants under high and low discount rates. As argued above, the gap between the two curves widens up to the

point of peak normal cost, at exit age 65. The uniform cost rate, n , is a weighted average of the individual rates, so it should rise by an amount that exceeds the (smaller) rise of the individual rates on the left side of Figures 1 and 2 and is less than the (larger) rise on the right side. And so it does: n rises by 6.1 percent (from 13.8 to 19.9 percent), while the individual normal costs rise by amounts close to zero for early departures, and up to 9 percent for departures at age 65.

What does this mean for the cross-subsidies? The cross-subsidies are the gaps (negative or positive) between the individual normal cost rates and the uniform rate. On the left side, the rise in the uniform rate exceeds the rise in individual rates, widening the gap. Conversely, on the right side, the individual rates rise by more than the uniform rate, widening the gap here, too. Thus, on both sides, we find an increase in the magnitude (absolute value) of the cross-subsidies provided and received. In other words, a drop in the discount rate increases the amount of redistribution, as measured by the cross-subsidies in normal cost rates. For example, the extreme points depicted in Figure 2 now represent cross-subsidies of -11.4 to +8.3 percent to percent, widening the previous range (-6.9 to +7.3), especially among the losers. On average, the losers provide cross-subsidies that widen from -2.6 percent of their income to -5.1 percent, while the winners receive cross-subsidies that rise from 2.5 percent of income to 3.1 percent. The winners' share of lifetime earnings rises with the drop in the discount rate (since they tend to serve longer), from 51 percent to 63 percent, so the zero-sum result on cross-subsidies still holds: $0.63 \times 3.1\% - 0.37 \times 5.1\% = 0.0\%$. Finally, taking the absolute values, we find that our measure of redistribution rises from 2.6 percent of total income to 3.8 percent ($0.63 \times 3.1\% + 0.37 \times 5.1\%$). This is the first of our two main results in this paper: *funding plans based on over-optimistic market return assumptions understate the degree of redistribution within cohorts, especially the impact on the losers.*

IV. CROSS-SUBSIDIES WITHIN AND BETWEEN COHORTS, WHEN THE FUNDING PLAN FAILS

Thus far, we have considered the distributional impact of lowering the assumed rate of return, as the normal costs rise non-uniformly, while the contribution rises uniformly. The cross-subsidies rise, but the redistribution remains within the cohort. We now consider the case where the assumed rate of return, r , is maintained at a high level, r' , while the “true” rate of return, r^* , is lower. Thus, the funding plan fails. Consequently, the cross-subsidies are not confined to within cohorts, but arise between cohorts. We consider two configurations, which may be interpreted as the short-run and long-run impact of maintaining $r = r'$ in the face of $r^* < r'$. In the short-run, contributions are maintained at the artificially depressed normal cost rate, r' , but in the long-run, as unfunded liabilities accumulate, contributions rise to a steady-state level, detailed below.

Short-Run Impact of Over-Optimistic Assumptions on Cross-Subsidies

Over-optimistic assumptions keep the calculated normal cost low, so in the short-run, contributions are low as well – that is arguably the unspoken motivation behind the assumption. Thus, the cross-subsidies provided and received by the current cohort are straight-forward, given by $(n^*_{es} - n')$, where n^*_{es} are the true individual normal costs and the contribution rate is artificially depressed at n' . In Figure 2, these would be represented by the gaps between the curves shown and the line for n' from Figure 1 (not shown in Figure 2). These can be decomposed as:

$$(10) \quad (n^*_{es} - n') = (n^*_{es} - n^*) + (n^* - n'),$$

where the first term is the cross-subsidy within the cohort and the second term is between cohorts. The weighted sum across the cohort of the first term is zero, as we have seen above. The second term – uniform across the cohort – is positive, representing the normal costs that are

accrued but not yet paid. This may be thought of as the cross-subsidy to be received by the current cohort from future cohorts, in current dollar terms.

Naturally, the low-balling of the contribution rate reduces the number of losers, to those who would otherwise be the biggest losers. We estimate that if the true return were 6.0 percent when CalSTRS assumed 7.5 percent, the losers would comprise about half the cohort, with only 14 percent of the present value of earnings. Their average cross-subsidy would be -10.5 percent of pay to the rest of the cohort, offset by receiving a cross-subsidy of $(n^* - n') = (19.9\% - 13.8\%) = +6.1$ percent of pay from future cohorts, for a net cross-subsidy of -4.4 percent. Conversely, the winners receive cross-subsidies of +1.7 percent from their cohort and +6.1 percent from the future, for a total cross-subsidy of +7.8 percent of pay. One may readily check that the within-cohort cross subsidies sum to zero ($0.14 \times -10.5\% + 0.86 \times 1.7\% = 0.0\%$) while the total cross-subsidies sum to +6.1% ($0.14 \times -4.4\% + 0.86 \times 7.8\% = +6.1\% = (n^* - n')$), the shortfall in contributions, to be met by future cohorts.

Steady-State Impact of Over-Optimistic Assumptions on Cross-Subsidies

Over time, over-optimistic assumptions lead to rising contributions, as unfunded liabilities grow and actuarial funding systems attempt to amortize them. Thus, each cohort, as a whole, receives lower and lower cross-subsidies from future cohorts, and, eventually, a reversal of the direction of cross-subsidies. Of course, it is not surprising that if an early cohort contributes less than the true normal cost, the shortfall must be made up by future cohorts paying more than the normal cost. That is, what began as current cohorts receiving cross-subsidies from future cohorts means that, when the future arrives, the current cohort is cross-subsidizing past cohorts. If that were the end of the story, this would simply be a transitory phenomenon, as

current cohorts pay off the unfunded liabilities accrued by prior cohorts. However, there is more to the story than that. As [Costrell \(2016b\)](#) showed, *this pattern of cross-subsidization from current to past cohorts persists in steady state: it does not abate, even asymptotically.*

Formally, with true returns r^* , the steady-state contribution rate is:

$$(11) \quad c^* = n^* + (c^p - n^*)(1 - f^*),$$

where f^* is the steady-state funded ratio (with liabilities evaluated at the true rate r^*), and c^p is the pay-go rate (as a percent of payroll). Thus, $c^* > n^*$ if $c^p > n^*$ and $f^* < 1$. The first condition is, in fact, the rationale for an actuarial pre-funding system, and will hold when r^* exceeds the payroll growth rate, g . In addition, the steady-state funded ratio $f^* < 1$ under conventional amortization formulas, when the assumed return r' exceeds the true return r^* . Indeed, the steady-state *measured* funded ratio, f'^* , is less than one, even as it inflates the true ratio f^* by discounting liabilities at r' when the true rate is r^* . As [Costrell \(2016b\)](#) shows:

$$(12) \quad f^* < f'^* = (r' - g) / \{ (r' - g) + (r' - r^*) [(1 + r') / (1 + g)]^N - 1 \} < 1 \text{ for } r' > r^* > g,$$

where N is the amortization period. Consequently, the steady-state contribution rate not only exceeds the cohort's low-balled normal cost n' , it also exceeds the true normal cost n^* .

Using data from the CalSTRS valuation report, together with a bit more of the steady-state math in [Costrell \(2016b\)](#), we can estimate c^* . First, we estimate c^p to fall in the range of 35 – 40 percent.¹⁴ Next, using (12) above, we calculate $f'^* = 56.8$ percent for, $r' = 7.5$ percent,

¹⁴ We estimate c^p two ways. First, we take a direct estimate from CalSTRS' reported ratio of benefits to payroll, 42.7 percent, and adjust that ratio downward, because the benefits currently paid reflect the old benefit formula rather than the one for new hires. Using CalSTRS' normal cost rates for these two benefit formulas (after netting out death and disability costs), we find $c^p = 36.3$ percent. Our second estimate takes CalSTRS' reported ratio of measured liabilities (at r') L' to payroll, adjusts that downward in the same way for new vs. old benefits, to 7.33, and applies the steady-state math of [Costrell \(2016b\), equation \(3\)](#), to find $c^p = n' + (L'/\text{payroll})(r'-g)/(1+g) = 40.3$ percent. Thus, we consider the range $c^p = 35 - 40$ percent. In the future, we plan to estimate c^p directly from CalSTRS' actuarial assumptions, in the same way we have estimated the normal cost rates.

$r^* = 6.0$ percent, $g = 3.75$ percent (CalSTRS' assumption), and an open amortization interval of 30. Finally, plugging these estimates, along with n' , into the result from [Costrell \(2016b\)](#),

$$(13) \quad c^* = n' + (c^p - n')(1 - f^*)/[1 - ((1+g)/(1+r'))^N],$$

we calculate the steady-state contribution rate c^* at 28 – 31 percent.¹⁵

Using this result, we can also back out of (11) estimates of the true funded ratio f^* at 44 - 48 percent. Thus, in steady-state the contributions will be augmented above true normal cost by over half of the gap between the pay-go and normal cost rates, $(c^p - n^*)(1 - f^*)$. This excess represents the steady-state cross-subsidy from the current cohort to past cohorts.

Figure 4 depicts the results. The individual normal cost rates, based on the “true” return of 6.0 percent, are reproduced from Figure 2, and the individual cross-subsidies are the gaps between the individual rates and the steady-state contribution rate c'^* . The key result here is that *all (or virtually all) individuals are losers, with negative cross-subsidies. The contribution rate exceeds the value of benefits even for those whose benefits are the most costly.*

The cross-subsidies can, again, be decomposed within and between cohorts:

$$(14) \quad (n^*_{es} - c^*) = (n^*_{es} - n^*) + (n^* - c^*).$$

If we take the lower bound estimate, $c^p = 35$ percent, such that $c^* = 27.8$ percent, only 2 percent of the cohort (with 2 percent of their earnings) enjoys benefits worth more than the contributions, receiving an average cross-subsidy of +0.9 percent. They receive a +8.8 percent subsidy from within the cohort, which is offset by -7.9 percent $((n^* - c^*) = 19.9 - 27.8$ percent) contributed to pay benefits of past cohorts. Conversely, the other 98 percent are losers, providing net cross-

¹⁵ By comparison, CalSTRS' 2015 contribution rate, based on $r' = 7.5$ percent, is 30 – 33 percent, and is projected to remain in that range to 2046. CalSTRS' 2016 valuation report, projects contributions rising to over 40 percent, based on $r = 7.0$ percent. Our estimates of the steady-state rate are based on factors that both lower the rate (as described in the note above) and raise the rate, by virtue of our lower steady-state funded ratio, persisting with assumed return of 7.5 percent while the “true” return $r^* = 6.0$ percent.

subsidies of -8.1 percent (-0.2 percent within the cohort and -7.9 percent to past cohorts. Again, the within-cohort cross subsidies sum to zero ($0.02 \times 8.8\% - 0.98 \times 0.2\% = 0.0\%$) while the total cross-subsidies sum to -7.9%, the excess contributions, to fund past cohorts ($0.02 \times 0.9\% - 0.98 \times 8.1\% = -7.9\% = (n^* - c^*)$).

V. CONCLUSION

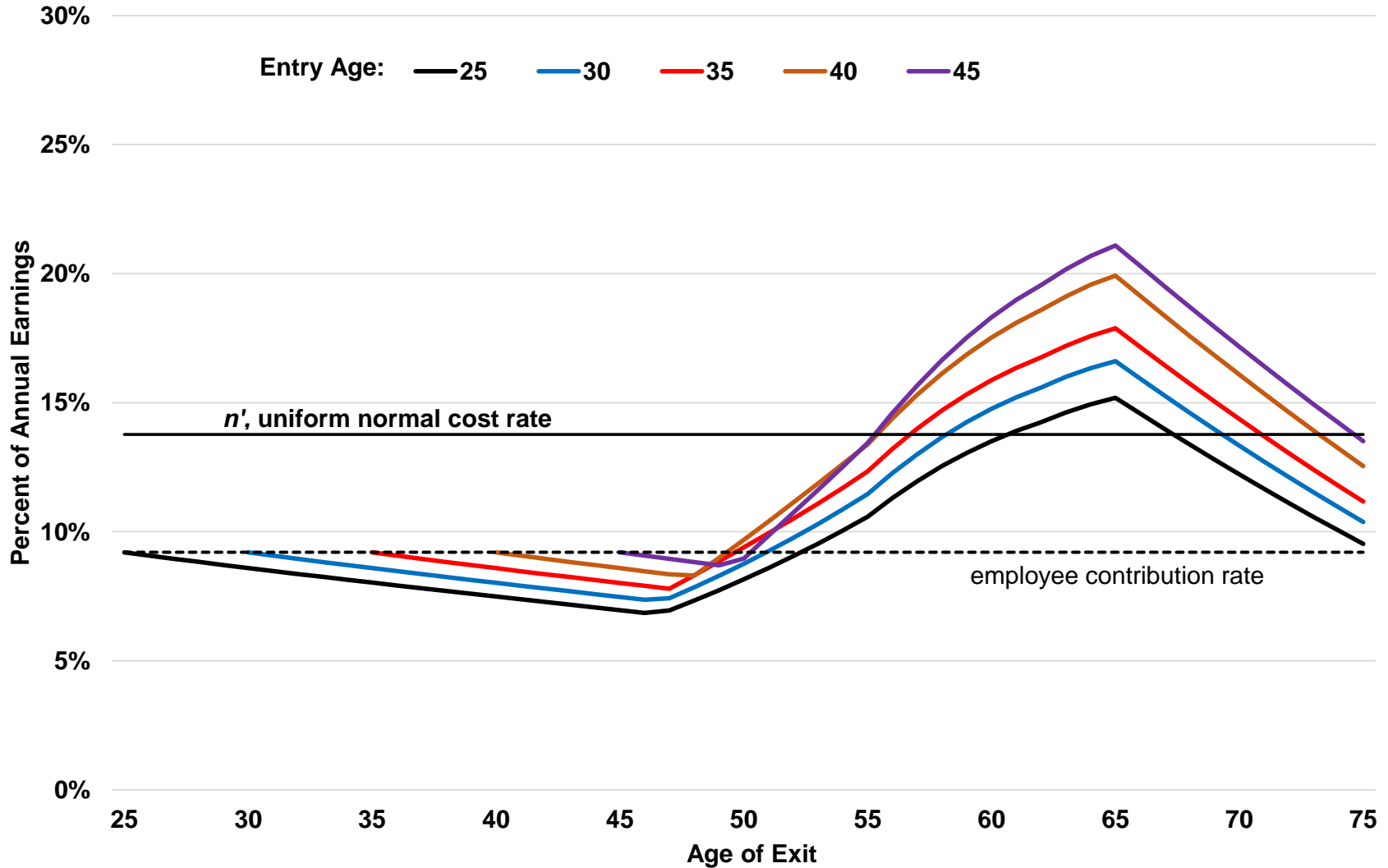
It is well-known that public pension plans exhibit substantial cross-subsidies, both within cohorts, e.g. from early leavers to those who retire at the “sweet spot” ([Costrell and Podgursky, 2010](#)), and across cohorts, through unfunded liabilities. However, the cross-subsidies within and across cohorts have never been provided in an integrated format. This paper provides such a framework, based on the gaps between normal cost rates for individuals (building on [Costrell and McGee, 2017](#)) and the uniform contribution rates for the cohort. Since the unfunded liabilities and associated cross-subsidies across cohorts derive from overly optimistic actuarial assumptions, we focus on the historically most important such assumption, the rate of return. We present two main findings. First, an overly optimistic assumed return understates the degree of redistribution within the cohort. Second (building on [Costrell, 2016b](#)), persisting with an overly optimistic assumed return leads to steady-state contribution rates that exceed the true normal cost (let alone the low-balled rate), i.e. cross-subsidies from the current cohort to past cohorts. Using the case of California, we show how that negative cross-subsidy can easily swamp all positive cross-subsidies within the cohort, as contributions exceed the value of benefits received by even the most favored individuals – those who retire at the “sweet spot.”

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Figure 1. Normal Cost Rate, by Entry Age and Age of Exit, $r' = 7.5\%$

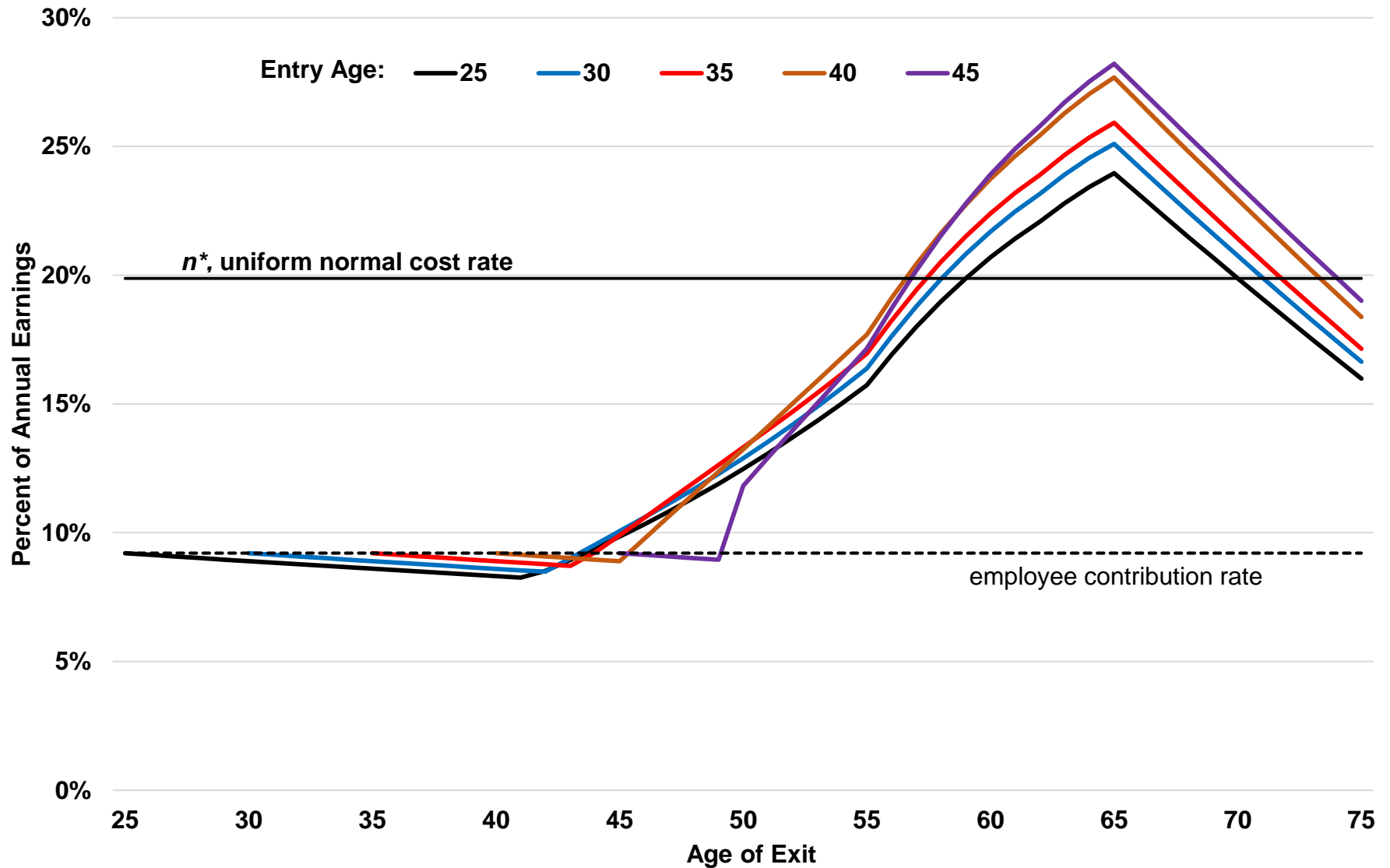
Estimated using 2015 CalSTRS assumptions and benefit formula for new hires, slightly modified



The curves depict n'_{es} , the annual contribution rate required to fund benefits of an individual entering at age e and exiting at age s . Variation in cost by age of exit is shown along each curve; variation by age of entry is shown across curves.

Figure 2. Normal Cost Rate, by Entry Age and Age of Exit, $r^* = 6.0\%$

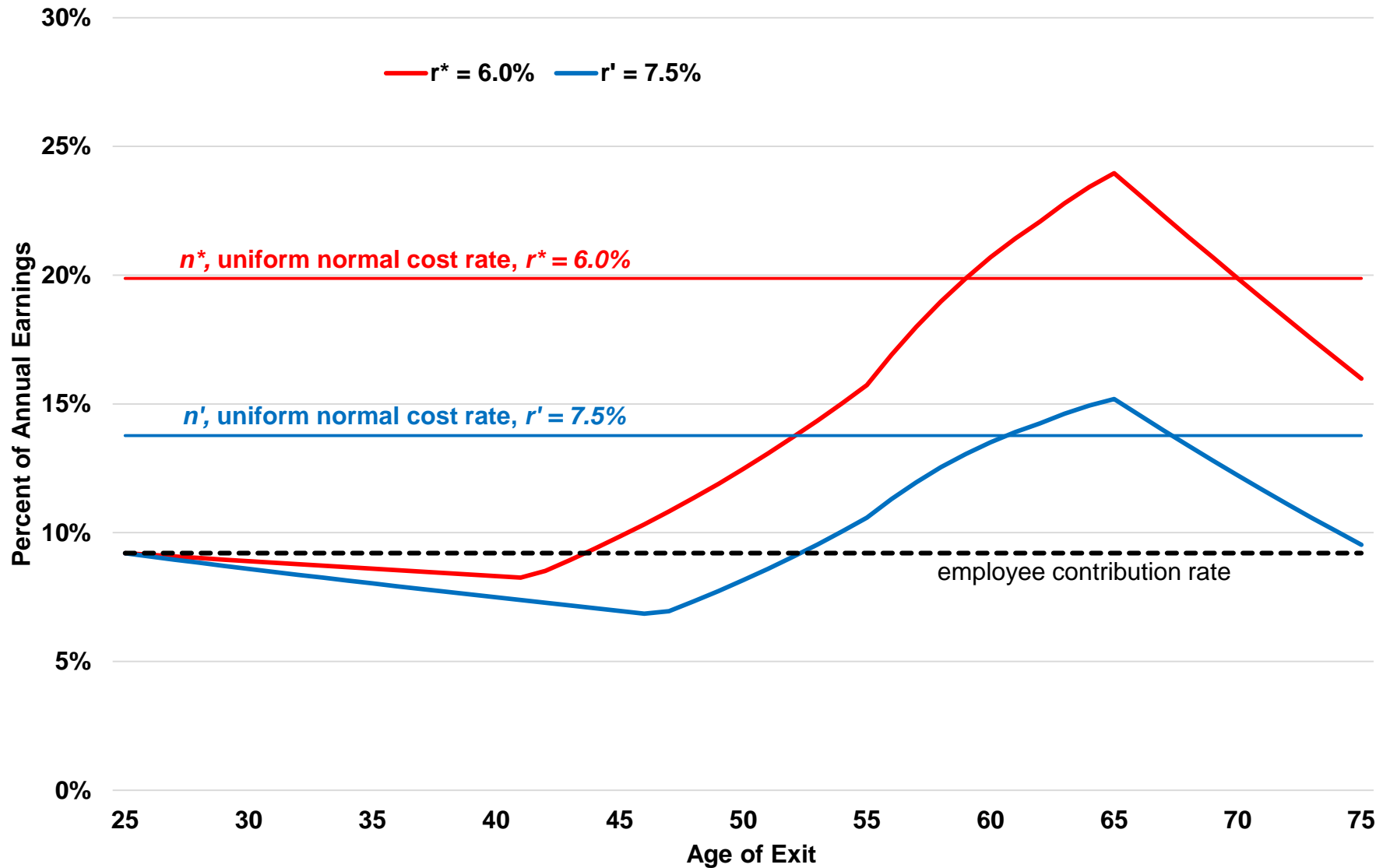
Estimated using 2015 CalSTRS assumptions and benefit formula for new hires, slightly modified



The curves depict n^*_{es} , the annual contribution rate required to fund benefits of an individual entering at age e and exiting at age s . Variation in cost by age of exit is shown along each curve; variation by age of entry is shown across curves.

Figure 3. Normal Cost Rate, Entry Age 25, $r^* = 6.0\%$ vs. $r' = 7.5\%$

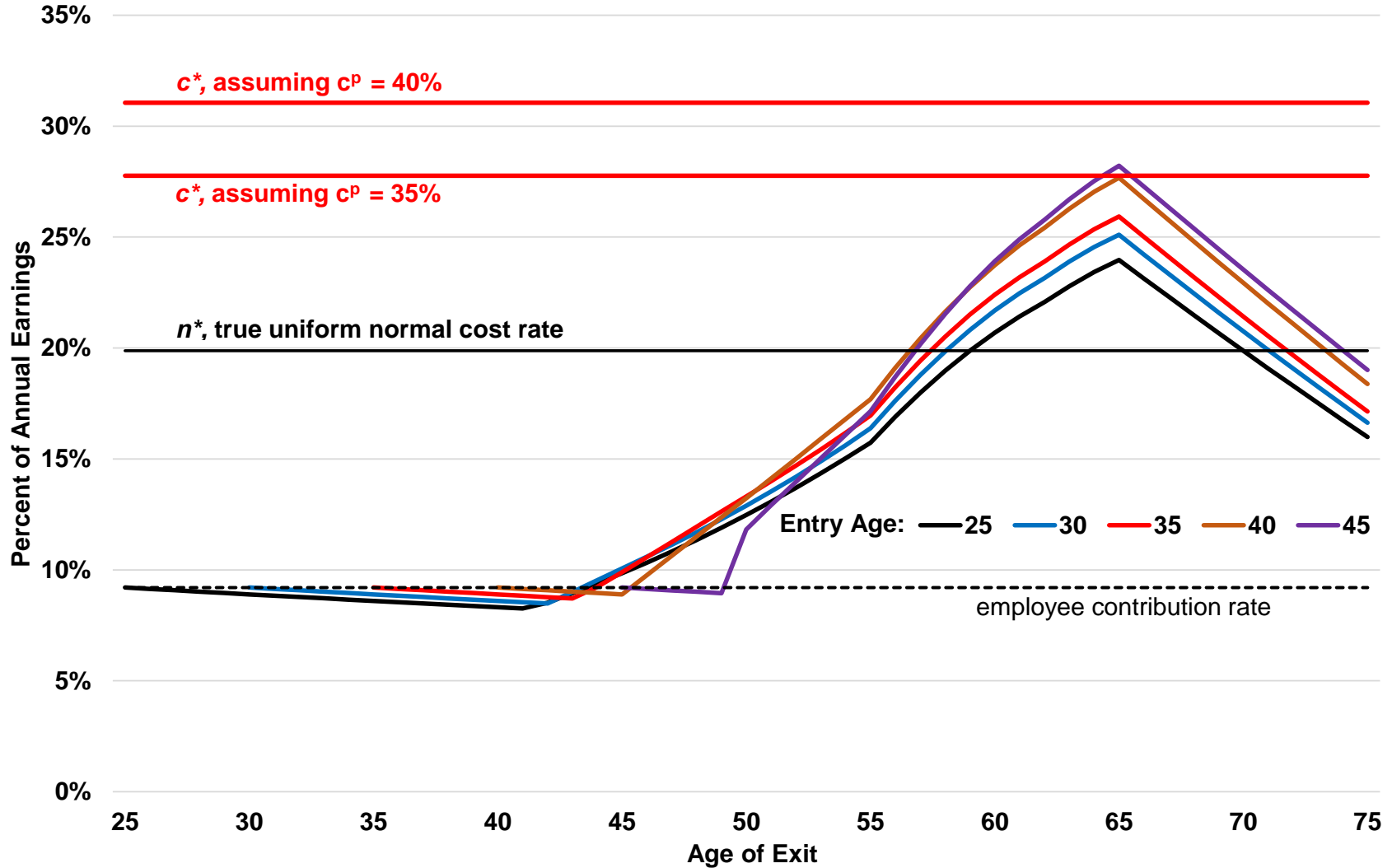
Estimated using 2015 CalSTRS assumptions and benefit formula for new hires, slightly modified



The curves depict $n^*_{25,s}$ and $n'_{25,s}$, the annual contribution rates required to fund benefits of an individual entering at age 25 and exiting at age s . Variation in cost by age of exit is shown along each curve; variation by assumed return shown across curves.

Figure 4. Steady-State Cross-Subsidies, $r^* = 6.0\%$, $r' = 7.5\%$

Steady-state contribution rate, c^* , derived using pay-go estimates $c^p = 35-40\%$ from CalSTRS data and Costrell (2016b)



The steady-state cross-subsidy for an individual entering at age e and exiting at age s is the gap between the curve and c^* line.
 The gap between the curve and n^* line is the within-cohort cross-subsidy and the gap between n^* and c^* is across cohorts.