CROSS-SUBSIDIZATION OF TEACHER PENSION COSTS:
THE IMPACT OF THE DISCOUNT RATE

Robert M. Costrell, Professor of Education Reform and Economics, University of Arkansas

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ABSTRACT: This paper builds on previous work (Costrell and McGee, 2017a) on the redistribution of teacher pension benefits, as measured by the wide variation in individual normal cost rates by age of entry and exit, and the associated cross-subsidies embedded in the funding plan. The further step taken here is to consider the impact of the discount rate on the degree of redistribution. We know that a reduction in the discount rate raises the normal cost of the plan, but the impact on the distribution of individual normal costs has not been previously explored.

There are two possible interpretations and motivations for this inquiry: (i) if the discount rate is the expected return, this analysis considers how the cross-subsidies vary as the expected return is reduced in funding plans, to better align with the market; and (ii) since the expected return includes a risk premium, netting that out of the discount rate (i.e. using the risk-free rate) tacks on a measure of the value of the benefit guarantee to the plan participants (Brown and Wilcox (2009), Novy-Marx and Rauh (2009), and, relatedly, Biggs (2011)). We know that the guarantee raises the value of benefits, as measured by the normal cost (Richwine and Biggs (2011)). In this paper I examine whether it raises the individual values uniformly or whether – and how much – it tilts the distribution of benefits.

Analytically, the impact is not obvious, as the impact of the discount rate varies with the age of exit for multiple reasons, and does not vary monotonically. Overall, however, using the illustration of California, I find that a lower discount rate raises the degree of redistribution substantially. Put differently, although the cross-subsidies embedded in the funding plan are quite significant, these understate the redistribution of benefits as the assumed return is reduced, and, dramatically more so, when the value of the guarantee is included.

KEYWORDS: teacher pensions

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I. INTRODUCTION AND SUMMARY

The funding plans for traditional teacher pension systems are built upon a highly uneven set of benefits, varying widely in value by age of entry and exit.¹ These inequities are masked by a uniform fringe benefit rate for pensions. For example, the annual contribution to the pension fund (employer and employee contributions taken together) may be 15 percent of each teacher’s salary. These “normal cost” contributions are designed to fund the future retirement benefits as they are earned,² for the system as a whole. However, the annual cost of benefits for individual teachers may deviate widely from this overall average, as shown previously (Costrell and McGee, 2017a, Costrell, 2018, Costrell and Fuchsman, 2018). As is well-established, the benefits for early leavers are of much lower value than for those who retire at the “sweet spot,” and, as shown previously, the contributions by or for the former effectively include a cross-subsidy to the latter. The further step taken here is to consider the impact of the discount rate on this system of redistribution. We know that a reduction in the discount rate raises the normal cost of the plan, but the impact on the distribution of individual normal costs has not been previously explored.

There are two possible interpretations and motivations for this inquiry: (i) if the discount rate is the expected return on the investment portfolio, this analysis considers how the cross-subsidies vary as the expected return is reduced in funding plans, to better align with the market;

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¹ This line of research dates to Costrell and Podgursky (2008, 2009, 2010a, 2010b).
² In addition, the employer makes payments for the unfunded liability – benefits earned in the past, but not funded. This is a very large problem, but is not the subject of this brief. The intergenerational cross-subsidies represented by these payments (Backes, et. al. (2016)) are a consequence of the failure to meet actuarial assumptions, particularly the return on investments (Costrell (2016a,b)). For analyses that incorporate cross-subsidies across generations that arise from the failure to meet assumed investment returns, see Costrell and McGee (2017b).
and (ii) since the expected return includes a risk premium, netting that out of the discount rate
(i.e. using the risk-free rate) tacks on a measure of the value of the benefit guarantee to the plan
participants (Brown and Wilcox (2009), Novy-Marx and Rauh (2009), and, relatedly, Biggs
(2011)). That is, we know that the guarantee raises the value of benefits, as measured by the
normal cost rate (Richwine and Biggs (2011)). In this paper I examine whether it raises the
individual values uniformly or whether – and how much – it tilts the distribution of benefits and
increases the cross-subsidies.

Analytically, the impact is not immediately obvious, as the impact of the discount rate on
the normal cost varies with the age of exit for multiple reasons, and the impact does not vary
monotonically. Overall, however, using the illustration of the California State Teachers
Retirement System (CalSTRS), I find that a lower discount rate raises the degree of
redistribution significantly. Put differently, although the cross-subsidies embedded in the
funding plan can be substantial, these understate the redistribution of benefits as the expected
return is reduced and, dramatically more so, when the value of the guarantee is included.

The plan of the paper is as follows. First, I will review the math of individual normal
costs and the associated cross-subsidies within cohorts embedded in the funding plan, under any
given discount rate used for that plan. I illustrate with the case of CalSTRS, using the plan’s
current expected return. I then compare it with the expected return held until recently, and
possible future assumed return. Next, I consider a risk-free rate; although it is unlikely to be
used for funding purposes, it does illustrate the wide variation in the value of benefits including
the benefit guarantee. I then turn to the mathematics of the distributional impact of the discount
rate, to better understand our results and help provide some intuition. Finally, in the conclusion,
I will briefly discuss potential policy implications, limitations to this work, and extensions to it.
II. INDIVIDUAL NORMAL COST RATES AND CROSS-SUBSIDIZATION

Pension plans calculate the normal cost rate at the aggregate level, to fund a cohort’s benefits as they accrue. Individual cost rates, based on age of entry and exit are implicitly embedded within the calculation (Costrell and McGee (2017a), Appendix), but they are not publicly reported. Specifically, consider an individual of type \((e,s)\), where \(e\) is the age of entry and \(s\) (for separation) is the age of exit. For each type \((e,s)\), one can identify an individual normal cost rate, \(n_{es}\), as a constant percent of salary over one’s career. This rate is calculated to generate a stream of contributions sufficient to fund the individual’s future benefits. That is, the present value (PV) of contributions must equal the PV of benefits.

Formally, for an individual of type \((e,s)\), we must have \(n_{es}W_{es} = B_{es}\), where \(W_{es}\) is the PV of earnings (so \(n_{es}W_{es}\) is the PV of contributions) and \(B_{es}\) is the PV of benefits (both evaluated at entry). It immediately follows that the individual cost rate is the ratio of the PV of benefits to that of earnings:

\[
(1) \quad n_{es} = \frac{B_{es}}{W_{es}}.
\]

This is the rate that, applied to the individual’s annual earnings over her career, would prefund her benefits. It represents the value of her benefits earned annually, as a percent of earnings – an individual fringe benefit rate for pensions (analogous to contributions to a retirement account).

If we compare individuals with different entry and exit ages, \((e,s)\), we find their cost rates, \(n_{es}\), vary widely. In general, for any given \(e\), \(n_{es}\) rises with \(s\), from some point after vesting up through a peak value retirement age. This is a manifestation of the well-known back-loading of benefits that favors long-termers under traditional pension formulas based on final average
salary, FAS (Costrell and Podgursky, 2009, 2010a). The variation in $n_{es}$ with $e$, for any given $s$, is less obvious, and can go either way.\(^3\)

Traditional pension plans levy a joint (employee plus employer) contribution rate, $n$, that is *uniform*, applied to all members of the cohort (of varying entry ages) throughout their careers (of varying length), calculated to fund the benefits of the whole entering cohort. Formally, denote the joint frequency of ages of entry and exit, $e$ and $s$, among entrants, as $p_{es}$. It can readily be shown that the uniform cost rate required to fund the cohort’s projected benefits is:\(^4\)

$$n = \frac{\sum_e \sum_s n_{es}(p_{es}W_{es})}{\sum_e \sum_s p_{es}W_{es}}.$$\(^2\)

This is the ratio of the PV of the cohort’s benefits\(^5\) to the PV of the cohort’s earnings: the same relationship we saw for the individual normal cost rate holds for the cohort as a whole. This expression also shows, importantly, that $n$ is a weighted average of individual normal cost rates $n_{es}$ across ages of entry and exit. The weights for $n_{es}$ are $(p_{es}W_{es})/(\sum_e \sum_s p_{es}W_{es})$, representing the share of type $(e,s)$ in the cohort’s PV of earnings.\(^6\)

The deviations $(n_{es} - n)$ are positive and negative, as the cost of funding any individual’s benefit exceeds or falls short of the cohort’s uniform contribution rate, $n$. They effectively comprise a system of cross-subsidies. Moreover, by the nature of averages, these cross-subsidies must add up to zero, when properly weighted, by shares of the cohort’s PV of earnings:

\(^3\) Later entrants with the same exit age have shorter service, so their pension and its PV, $B_{es}$, is lower, but so is that of their earnings, $W_{es}$. Thus, the ratio, $n_{es} = B_{es}/W_{es}$, can rise or fall, over different ranges of $s$, discount rates (compare Figures 1 and 4 below), and benefit formulas (e.g. compare CalSTRS, below and Massachusetts, Costrell and Fuchsman, 2018). Another way of seeing the ambiguity is to note that for any given exit age, the normal cost rate varies with (i) the starting pension as a percent of FAS; and (ii) FAS relative to cumulative earnings. For older entrants, with shorter service, the starting pension is a lower percent of FAS, which reduces normal cost. But their FAS is higher relative to cumulative earnings (since it is a shorter stream), which raises normal cost.

\(^4\) It can be shown that $n$ applies not simply to a single entering cohort, but to any cohort, past or present, or the full set of such cohorts working their way over time through the workforce, under a given benefit formula and set of actuarial assumptions (Costrell and McGee (2017a)).

\(^5\) Substituting $n_{es} = B_{es}/W_{es}$ into the numerator gives $\sum_e \sum_s p_{es}B_{es}$.

\(^6\) These are not the exact weights used in actuarial practice, but are consistent with the approach (see Costrell and McGee (2017), Appendix).
\[
\sum_e \sum_s (n_{es} - n)(p_{es}W_{es}) / (\sum_e \sum_s p_{es}W_{es}) = 0.
\]

The negative cross-subsidies provided by the losers fund the positive cross-subsidies enjoyed by winners. I will illustrate the system of cross-subsidies under CalSTRS’ current plan, followed by an examination of how this system of redistribution is affected by the discount rate.

**III. INDIVIDUAL NORMAL COST RATES FOR CALSTRS**

I now apply these concepts specifically to the California State Teacher Retirement System plan. I estimate the individual normal cost rates, \( n_{es} = B_{es}/W_{es} \), for all entry and exit ages, \( e,s = 20, \ldots, 75 \). I base the calculations on the CalSTRS actuarial assumptions (slightly modified, as explained below) and benefit formula.\(^7\)

Benefits can be in the form of a pension or refund of employee contributions.\(^8\) If a teacher takes the refund she forgoes any future pension and receives, instead, the cumulative value of the employee (but not employer) contributions, with accumulated interest at the rate set by CalSTRS. Teachers who leave before vesting, without the expectation of returning and becoming eligible for a pension, would certainly take the refund because it is the only benefit to which they are entitled. Teachers who leave after vesting, but too young to draw a pension, may either take the refund or leave the money in the fund to draw a pension in the future, upon reaching an eligible age. Finally, teachers who leave service and are eligible for an immediate

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\(^7\) The actuarial assumptions cover wage growth, investment returns, exit rates, and mortality rates. These assumptions are drawn from the 2016 annual valuation report (CalSTRS, 2017), based on the most recent 5-year experience study (CalSTRS, 2016). (The actuarial assumptions used in Costrell and McGee (2017a) were drawn from the 2015 valuation report, based on the prior 5-year experience study.) The benefit formula is delineated in the valuation report and the member handbook (CalSTRS, 2018). This includes the retirement eligibility conditions, age-specific multipliers, cost of living adjustments (COLA), employee contribution rate, and interest rate on refunds.

\(^8\) I leave aside death and disability benefits, which comprise about 5 percent of normal cost, less than 1 point.
pension, may still choose the refund, although it is generally not financially prudent to do so. I assume that teachers choose the refund or pension to maximize the PV of their benefits.\(^9\)

If a teacher takes the pension, \(B_{es}\) is the PV of the stream of pension payments, weighted by her survival probabilities, discounted to entry. The payments begin with a starting pension equal to an age-specific multiplier \(\times\) years of service \((s – e)\times\) final average salary (FAS, last 3 years), augmented annually with a 2.0 percent simple COLA. Specifically, I consider the “2% at 62” program for new hires (since 2013), with multipliers ranging from 1.16 percent at age 55 to 2.0 percent at 62 and 2.4 percent at 65, after 5-year vesting.\(^{10}\) For example, a 25-year-old entrant working to 65 retires with a starting pension of \(40 \times 2.4 = 96\) percent of FAS. This formula, together with CalSTRS actuarial assumptions, allows us to calculate the PV of benefits, relative to the PV of wages, \(n_{es} = B_{es}/W_{es}\), the annual contribution rate required to fund the benefits of an individual entering at age \(e\) and exiting at age \(s\).

**Variation in Normal Cost Rates By Age of Entry and Exit**

We first consider the variation of normal cost rates under the current assumed return of \(r = 7.0\) percent.\(^{11}\) Figure 1 depicts the normal cost rates, \(n_{es}\), for selected ages of entry (representative of all ages) and all exit ages. The variation is wide, from 7.3 percent to 24.9 percent (the full range, for entry ages not shown, is 6.8 to 27.1 percent). The pattern for any given entry age (e.g., age 25) is depicted along each curve, as the exit age varies. Prior to vesting, and for some years beyond, the benefit is the refund of employee contributions. The

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\(^9\) CalSTRS assigns probabilities of taking the refund which may not maximize PV. Our modified assumption eliminates a precipitous drop in the individual normal cost rate upon vesting, due to suboptimal cash-outs.

\(^{10}\) Vested employees who withdraw before age 55 but do not cash out must defer the pension to at least age 55 and we assume they collect then. CalSTRS assumes they defer to age 62. Our modification eliminates a discontinuity in the individual normal cost rate between ages 54 and 55 that arises for lower discount rates.

\(^{11}\) The assumed return has been cut from 7.5 percent through the 2015 valuation (the valuation used in Costrell and McGee (2017a)) to 7.25 percent for the 2016 valuation and 7.00 percent thereafter.
normal cost rate, therefore, starts at the employee contribution of 10.2 percent:12 each curve begins at the dashed horizontal line representing that rate. The cost rate then gently declines, falling slowly below the employee contribution rate. That is because the interest credit of 3.0 percent is below the fund’s assumed return, 7.0 percent. The contribution rate needed to cover the refund falls as this difference accumulates.

At a certain point, the pension becomes more attractive than the refund. A 25-year-old entrant reaches that point at age 44; at this age the pension would still be deferred, but exceeds in PV the value of the employee refunds. Beyond that point, the normal cost rate rises as the deferral becomes shorter, and then, beyond age 55, there is no deferral, but the normal cost rate continues to rise as the age-specific multiplier grows. Each year of delayed retirement beyond 55 is a year of forgone pension payments, but prior to age 65, the growth in the multiplier outweighs this effect. After age 65 the multiplier stops growing, and the normal cost declines, due to the decreasing number of years the pension will be paid. This pattern is reflected in Figure 1 along each curve, corresponding to any given entry age. In addition to the variation within entry-age cohorts, Figure 1 also depicts the (vertical) variation across entry ages for the same exit age, which widens the overall spread in individual normal cost rates.

**Cross-Subsidy Rates and the Degree of Redistribution**

The wide variation among individual cost rates contrasts with the uniform contribution rate, $n$. As shown in (2), that is the weighted average of the individual cost rates, $n_{es}$, that will fund the benefits of each cohort, past and present, taken as a whole, under the current benefit formula and assumed return. I calculate $n$ to be 17.6 percent of pay, depicted in Figure 1 as the

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12 The rate has been 9.2 percent, but is expected to rise one percent, effective July 1, 2018 (CalSTRS, 2018, p. 5).
solid horizontal line.\footnote{CalSTRS (2017) calculates the normal cost rate for new hires as 17.8 percent at \( r = 7.00\% \) (and 16.7 percent at \( r = 7.25\% \)). Netting out death and disability benefits (about 1.0 percent), these estimates are slightly lower than mine, consistent with my assumption that there are no sub-optimal cashouts.} The deviations of individual cost rates from \( n \) represent the cross-subsidy rates, \((n_{es} - n)\). Those above the line receive cross-subsidies from those below. For example, the extreme points depicted for \( n_{es} \), of 7.3 and 24.9 percent, represent cross-subsidies of \(-10.3\) and \(+7.4\) percent of pay.\footnote{For points not depicted, the range is \(-10.8\) to \(+9.5\) percent.} These cross-subsidies are built into the funding plan. For those individuals below the solid line, the plan is counting on using some or all of the employer contributions – plus, for many, part of the employee contributions – along with some or all of the assumed returns to help finance benefits of those above the line.

Using the joint frequencies of entrants, \( p_{es} \), and their shares of lifetime earnings, \((p_{es}W_{es})/(\sum_{e}\sum_{s}p_{es}W_{es})\), we can calculate a few summary statistics. Those who provide the cross-subsidies (those below the line in Figure 1) comprise 61 percent of entrants\footnote{Rhee and Fornia (2016, 2017) argue that prior entrants who are no longer in the workforce should be excluded when counting winners and losers. But as explained in Costrell and McGee (2017a), this results in “survivorship bias” toward winners. As a result, the losses left behind by prior leavers are excluded, such that the cross-subsidies do not sum to zero. In other words, the funding math simply does not add up.} and account for 36 percent of their lifetime earnings;\footnote{Those with lower normal cost rates (negative cross-subsidies) tend to be early leavers with shorter earnings streams, so smaller shares of the cohort’s PV of earnings.} those who receive the cross-subsidies are the remainder. Taken together, the losers provide cross-subsidies that total \(-4.8\) percent of their lifetime earnings. That is the average cross-subsidy rate for those below the line (weighted by shares of lifetime earnings). The winners receive cross-subsidies that average \(+2.7\) percent. One can readily verify the zero-sum result from (3): \(0.64 \times 2.7\% - 0.36 \times 4.8\% = 0.0\%\). Thus, in all, taking absolute values of the cross-subsidies, \textit{3.4 percent of total income is redistributed}, about one-fifth of the total normal cost, and almost half the employer contribution.\footnote{While the uniform employer contribution rate is 7.4 percent, the losers receive, on average, employer funded benefits worth only 2.6 percent, while those of the winners’ are worth 10.1 percent.}
III. THE IMPACT OF THE DISCOUNT RATE ON CROSS-SUBSIDIES FOR CALSTRS

Reductions in the Assumed Return

The analysis above was based on \( r = 7.0 \) percent, the assumed return to be reflected in the 2017 valuation. This represents a reduction from 7.5 percent in the 2015 valuation and 7.25 percent in 2016. In reducing its assumed return from 7.5 percent to 7.0 percent, CalSTRS has, of course, raised its normal cost rate for new hires, \( n \) (about 2.1 percentage points by my estimate). The point here, however, is that the degree of redistribution also rose, from 3.0 percent of total income to 3.4 percent.

To illustrate the distributional impact of further rate reduction, consider what the system would look like if \( r \) were reduced to \( r = 6.0 \) percent (comparable to recent 10-year rolling averages). The individual normal cost rates are depicted in Figure 2. All the normal cost rates are increased from those depicted in Figure 1: with lower assumed investment returns, contributions must be higher to fund the benefits. This much is well-known. What is perhaps less widely understood is that a drop in the assumed return will increase the degree of redistribution embedded in the funding plan. Stated alternatively, an over-optimistic assumed return not only underfunds the plan, but also understates the degree of redistribution.

Specifically, we are interested in how the impact of \( r \) on the normal cost rates \( n_{es} \) varies, since that will determine the impact on the cross-subsidies and the degree of redistribution. If all \( n_{es} \) were to rise by the same amount with a drop in \( r \), then \( n \) would rise by approximately that amount;\(^{18}\) the cross-subsidies \((n_{es} - n)\) would remain nearly unchanged, and so would the degree of redistribution. This is not the case. The non-uniformity is illustrated in Figure 3, which depicts \( n_{es} \) for 25-year-old entrants as the assumed return drops. The gap between the two curves

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\(^{18}\) There would be some small effect from the change in weights, induced by the change in \( r \).
widens up to the point of peak normal cost, at exit age 65. This holds more generally for the various entry ages depicted in Figures 1 and 2. Figure 4 illustrates this point directly, depicting the rise in normal cost rates, by entry and exit age, as \( r \) drops from 7.0 to 6.0 percent.

The uniform cost rate, \( n \), is a weighted average of the individual rates, so it should rise by an amount that exceeds the (smaller) rise of the individual rates on the left side of Figure 4 and is generally less than the (larger) rise on the right side. And so it does: \( n \) rises by 5.2 percent (from 17.6 to 22.8 percent), while the individual normal costs rise by amounts close to zero for early departures, and up to 7 percent for departures at age 65.

What does this mean for the cross-subsidies? The cross-subsidies are the gaps (negative or positive) between the individual normal cost rates and the uniform rate. The rise in the uniform rate exceeds the rise in individual rates on the left side of Figure 4, widening the gap. Conversely, on the right side, the individual rates generally rise by more than the uniform rate, widening the gap here, too. Thus, on both sides, we find an increase in the magnitude (absolute value) of the cross-subsidies provided and received. In other words, a drop in the assumed return increases the amount of redistribution, as measured by the cross-subsidies in normal cost rates. For example, the extreme points depicted in Figure 2 now represent cross-subsidies of \(-14.4\) to \(+7.5\) percent, widening the previous range \((-10.3\) to \(+7.4\)), especially among the losers.\(^{19}\) On average, the losers provide cross-subsidies that widen from \(-4.8\) percent of their income to \(-6.8\) percent, while the winners receive cross-subsidies that rise from \(2.7\) percent of income to \(3.4\) percent. The zero-sum result on cross-subsidies still holds (with winners’ share of lifetime earnings now at 0.67), and, finally, taking the absolute values, we find that our measure of redistribution rises from \(3.4\) percent of total income to \(4.5\) percent \((0.67 \times 3.4\% + 0.33 \times 6.8\%)\).

\(^{19}\) For points not depicted, the range widens from \((-10.8\) to \(+9.5\)) to \((-14.8\) to \(+8.8\)).
The Distribution of Risk-Free Benefits

The critique of actuarial discount rate practice by the finance economics field (Brown and Wilcox (2009), Novy-Marx and Rauh (2009), and Biggs (2011)) distinguishes between the use of the assumed rate of return for determining contributions and the proper discount rate for reporting risk-free liabilities. Thus, quite aside from the contribution policy, on the part of employers, (Richwine and Biggs (2011)) have made the important point that from the viewpoint of the teachers, the value of the benefit guarantee is much understated by using a discount rate with a risk-premium. Using something close to the risk-free rate properly values the benefit at a much higher rate than the contributions, based on the expected returns on risky assets. This is a separate matter from the debate that continues in the public discourse on whether the cost of market risk to employers (and taxpayers) is obviated by public sector immortality (the alleged “free lunch” of time diversification) or whether that fallacy and others simply obfuscate the unreported cost of risk that is borne by the employers, to the benefit of the teachers. Ironically, both sides of that debate on the cost of risk to employers – the defenders and critics of current actuarial practice for public defined benefit plans – agree on the value to employees of the benefit guarantee (see, for example, Rhee and Fornia, 2017).

Thus, leaving aside the question of employer contributions and their redistribution, let us simply look at the distribution of individual benefits, including the value of the guarantee, annualized as a percent of earnings. This is the same as \( n_{es} \), but evaluated at something close to the risk-free rate, which I take to be 4.0 percent. Plans may choose to fund these benefits with risky assets and correspondingly lower contributions – a subject of debate elsewhere – but, again, both defenders and critics of traditional pension plans agree that the benefit guarantee is of great value. Figure 5 illustrates how great that value is, depending on age of entry and exit. The
average annualized value is nearly 40 percent, and the spread is enormous, ranging from about 10 percent (almost the value of employee contributions, for those taking refunds) to nearly 50 percent for those retiring at 65.\textsuperscript{20} In other words, the net value of employer-guaranteed benefits ranges from 0 to 40 percent. One-third of entrants will receive benefits worth at least 30 percentage points above the employee contribution, and two-fifths will receive benefits that are less than one point above the employee contribution, or slightly below it.

I refrain here from referring to cross-subsidies in contributions, since I am not identifying the average $n$ as a contribution rate (the employer cost of bearing risk may be borne in other ways, e.g. fluctuations in contributions). However, analogous to the cross-subsidies described above, the range of deviations between individual and average benefits is $-29.6$ percent to $+10.5$ percent. The average benefit of the losers lies below the mean by 11.0 percent of pay, and that of the winners is 5.7 percentage points above. Thus, our measure of redistribution rises to 7.5 percent of income.

\textbf{IV. THE ANALYTICS OF INDIVIDUAL NORMAL COST RATES AND THE DISCOUNT RATE}

It is well-known that a drop in the discount rate raises the overall normal cost rate, $n$; this is regularly reported in valuation reports when the assumed rate of return is reduced. Since we are interested in the impact on the distribution of individual normal cost rates, we back up and first examine more closely why a drop in $r$ raises the individual rates, $n_{es}$, so that we can then go on to examine how the impact varies across individuals. In general terms, the first point is simple: a fall in the discount rate raises the individual rates, $n_{es} = B_{es}/W_{es}$, because the sequence

\textsuperscript{20} Here we see that for any given exit age, $n_{es}$ falls with entry age, the opposite pattern from that with higher discount rates, as depicted in Figures 1 and 2, as mentioned in a note above.
of benefits follows in time the sequence of earnings. As a result, the PV of individual benefits, $B_{es}$, rises proportionally more than that of earnings, $W_{es}$.

Introducing the discount rate $r$ explicitly, we have, for the denominator of $n_{es}$:

(4) $W_{es} = \sum_{e=0}^{s}(1 + r)^{(e-a)}w_{a|e}$,

where $w_{a|e}$ is the wage at age $a$, given entry at age $e$. For the numerator of $n_{es}$, we have:

(5) $B_{es} = \sum_{a>s}(1 + r)^{(e-a)}b(a|e,s)f(a|s)$,

where $b(a|e,s)$ is the payment (pension or refund) at age $a$, given entry and exit ages $e$ and $s$, and $f(a|s)$ is the survival rate to age $a$, conditional on survival to exit age $s$. It is then straightforward to show that the proportional increase in $W_{es}$ as $r$ is reduced – the negative of the log derivative with respect to $r$ – is a weighted average of terms $(a-e)/(1+r)$, where $a$ runs from $e$ to $s$.

Similarly, the proportional increase in $B_{es}$ is another weighted average of terms $(a-e)/(1+r)$, where $a$ runs beyond $s$. That is, these are weighted averages of time since entry (divided by $(1+r)$). Thus, each term of the latter average exceeds each term of the former, so $B_{es}$ rises proportionally more than $W_{es}$, as stated above, and $n_{es}$ rises as $r$ drops.

This much is simply a formalization of the general point that $n_{es}$ rises as $r$ drops because of the non-overlapping time patterns in earnings and benefits. But (4) and (5) are potentially pertinent to the paper’s main point, which is logically subsequent: how the response to $r$ varies across individuals, particularly by $s$, resulting in a greater degree of redistribution. In effect, having examined the first derivative of $n_{es}$ with respect to $r$, we are now interested in the cross-partial with respect to $r$ and $s$. That is, we would like further insights into the pattern in Figure 4.

The first portion of Figure 4, where the benefit is taken as a refund, is simple to understand. As discussed earlier, at $s = e$, $n_{es}$ equals the employee contribution rate, independent of $r$, and as $s$ increases, the gap between $n_{es}$ and the employee contribution rate grows due to the
accumulating difference between \( r \) and the interest on refunds. As \( r \) is reduced, that difference narrows and \( n_{es} \) draws closer to the employee contribution rate. Thus, the impact on \( n_{es} \) of reductions in \( r \) increases from zero, at \( s = e \), to larger impacts as \( s \) increases. This is relatively easy to show analytically, but would be overly formalistic, since the intuition is fairly obvious.

The more difficult pattern to explain with analytic precision is the rest of Figure 4, where the benefit is taken as a pension. Figure 4 seems to indicate that the impact on \( n_{es} \) of reducing \( r \) rises with \( s \) so long as \( n_{es} \) itself rises with \( s \) (i.e. up to age 65) and falls with \( s \) thereafter. Why is this the case? As stated above, the impact of \( r \) on \( n_{es} \) arises from the difference between the weighted averages of time since entry for benefits and earnings, over the ranges of \( a \) above and below \( s \). This difference gives us the proportional increase in \( n_{es} \) with a drop in \( r \). The impact of \( r \) on \( n_{es} \) as measured in percentage points of earnings (i.e. the impact depicted in Figure 4) is \( n_{es} \times (\text{the difference in weighted averages})/(1+r) \). Thus, the effect of \( s \) on that impact works both through its effects on the difference in weighted averages and through its effect on \( n_{es} \) itself. The former effects are complex (\( s \) appears in both summation limits in (4) and (5), and similarly in their log derivatives with respect to \( r \)), and \( s \) also appears in the weights pertaining to benefits, through \( b(a|e,s)f(a|s) \). However, it appears empirically, in the case of CalSTRS, that these effects are swamped by the impact of \( s \) on \( n_{es} \) itself. In other words, it is the back-loading of benefits (up to age 65) that gets magnified as \( r \) drops, and, beyond 65, when further years of forgone pension payments reduce \( n_{es} \), so, too does \( s \) reduce the impact of \( r \) on \( n_{es} \). On balance, it appears that the overall back-loading of benefits under traditional FAS plans, which generates

\[ \text{Raising } s \text{ shifts both ranges toward higher values of } (a-e)/(1+r) \text{ (dropping a term at the low end for the sum above } s \text{ and adding a term at the high end for the sum below } s), \text{ with no obvious effect on the difference. That is, for given sets of weights, the effect of raising } s \text{ on the different time patterns of earnings and benefits is ambiguous.} \]
much of the redistributive cross-subsidies at issue, is magnified by reductions in the discount rate, and so is the redistribution.

V. CONCLUSION

The distinguishing characteristic of traditional FAS pension plans, such as CalSTRS, is that the benefit is delinked from contributions, unlike cash balance or other account-based plans (discussed briefly below). Some individuals receive benefits that cost more than the contributions made by or for them, and some receive less. Following on earlier work, I measure the value of individual benefits as the annual contributions required to fund them, as a percent of pay, which embed a system of hidden cross-subsidies, varying by age of entry and exit. The point of this paper is that the extent of this redistribution rises with a drop in the discount rate. Thus, if the assumed return is reduced, not only does the overall normal cost rate rise, the individual normal cost rates rise unevenly, tilting more toward long-termers. Stated conversely, if the assumed returns are over-optimistic, keeping contributions low, then the redistribution of those contributions is also understated. If we include the value of the Defined Benefit guarantee by using a risk-free discount rate (as argued by Richwine and Biggs (2011)), the tilt toward long-termers becomes quite dramatic.

We illustrate this pattern by considering the CalSTRS plan for new hires at its 2017 assumed return of 7.0 percent (down from 7.5 percent in 2015) and a hypothetical future value of 6.0 percent. The range of cross-subsidies depicted in the Figures above widens from (–10.3 to +7.4) percent of pay to (–14.4 to +7.5) percent. The average loss of those providing the cross-subsidies rises from 4.8 percent of their lifetime earnings to 6.8 percent, and degree of redistribution rises from 3.4 percent of total income to 4.5 percent.
Adding in the value of the DB guarantee, at a risk-free rate of 4.0 percent, widens the range of individual benefits, and the deviations from the (now much higher) average benefit rate of nearly 40 percent. These deviations range from (−29.6 to +10.5) percent. The average benefit of the losers lies below the mean by 11.0 percent of their pay, and our measure of redistribution rises to 7.5 percent of total income.

What are the policy implications of this analysis? At the very least, any good policy should be transparent. Where traditional FAS plans are employed, the system of hidden cross-subsidies should be laid bare. The uniform contribution rate, designed for funding purposes, masks the wide variation in individual cost rates. These rates can be readily calculated, by age of entry and exit, as a byproduct of the annual actuarial valuations, and should be made publicly available, so that members can better understand how their plan may affect them. Moreover, as both defenders and critics of traditional plans agree, the value of a defined benefit includes the value of the guarantee; the dramatically wide individual distribution of that benefit can be readily calculated and should be disclosed.

There is reason to go further, by reducing the actual variation in benefits.²² The most efficient way of reducing the variation is through an account-based system, such as a cash balance (CB) or defined contribution (DC) plan. A CB plan is a defined benefit plan, in which each individual’s benefit is directly tied to a retirement account balance (to be annuitized or drawn down). That balance is equal to the cumulative value of employee contributions and employer contribution credits (a bookkeeping entry), plus accumulated interest credits. In the baseline, hypothetical case where the interest credit equals the discount rate, and the employer contribution credit is uniform, that contribution credit is the employer-funded benefit, transparent

²² As argued elsewhere, there seems to be little evidence that such a wide distribution of benefits effectively or efficiently serves human resource goals. For a good summary of the research, see Koedel and Podgursky (2016).
to all. There are no cross-subsidies, as benefits accrue smoothly in tandem with contributions at a constant percent of pay, and, moreover, it is independent of the discount rate. In practice, employer contribution credits can increase with service, the guaranteed interest credit is typically less than the assumed return, and market risk can be shared.23

It is worth restating the caveat that this paper is restricted to cross-subsidies within cohorts, i.e. assuming actuarial assumptions hold, especially the assumed return. The gap in recent years between actual and assumed returns has led to unfunded liabilities, which generate large cross-subsidies across generations. Indeed, related work on CalSTRS cited above, aimed at an integrated treatment of cross-subsidies across and within cohorts, finds that virtually all current entrants can easily be losers, by virtue of the steady-state amortization payments to cover benefits of previous cohorts, if the assumed return is held somewhat higher than the actual return. It should also be noted that CB plans – like other, more traditional DB plans, and unlike DC plans – can also run up unfunded liabilities and generate cross-subsidies across generations.

23 For example, under Kansas’ Tier 3 CB plan (Schmitz, 2016; KPERS, 2017) – the nation’s first such plan covering teachers – the employer match (to the employee’s contribution of 6 percent) rises from 3 percent of pay for years 1-4 of service to 6 percent of pay for years 24 and beyond. Typical of other CB plans, the employers’ actual contribution rate is less than the notional contribution credit, because the plan’s assumed return exceeds the assumed interest credit. I calculate that the individual employer normal cost rate, averaged over one’s years of service, for providing these credits would range from 3.0% to 4.7%, with interest credits equal to the assumed return of 7.75%. Under the assumed interest credit of 6.25% (including expected “dividends” over the guaranteed rate of 4.0%), the range in individual employer normal cost rates is only 3.0% to 3.7%. Either way, the range is much narrower than under FAS systems.
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Figure 1. Normal Cost Rate, by Entry Age and Age of Exit, $r = 7.0\%$

Estimated using current CalSTRS assumptions and benefit formula for new hires, slightly modified

The curves depict $n_{es}$, the annual contribution rate required to fund benefits of an individual entering at age $e$ and exiting at age $s$. Variation in cost by age of exit is shown along each curve; variation by age of entry is shown across curves.
Figure 2. Normal Cost Rate, by Entry Age and Age of Exit, $r = 6.0\%$

Estimated using current CalSTRS assumptions and benefit formula for new hires, slightly modified.

The curves depict $n_{es}$, the annual contribution rate required to fund benefits of an individual entering at age $e$ and exiting at age $s$. Variation in cost by age of exit is shown along each curve; variation by age of entry is shown across curves.
Figure 3. Normal Cost Rate, Entry Age 25, $r = 6.0\%$ vs. $r = 7.0\%$

Estimated using current CalSTRS assumptions and benefit formula for new hires, slightly modified.

The curves depict $n_{25,s}$, the annual contribution rates required to fund benefits of an individual entering at age 25 and exiting at age $s$. Variation in cost by age of exit is shown along each curve; variation by assumed return shown across curves.
The curves depict $\Delta n_{es}$, the rise in annual cost to fund benefits of an individual entering at age $e$ and exiting at age $s$, as $r$ falls from 7.0% to 6.0%. Variation in $\Delta n_{es}$ by age of exit is shown along each curve; variation by age of entry is shown across curves.

Figure 4. Rise in Individual Normal Cost Rates, $r = 6.0\%$ vs. $7.0\%$
Estimated using current CalSTRS assumptions and benefit formula for new hires, slightly modified

Entry Age:  
- 25  
- 30  
- 35  
- 40  
- 45
Figure 5. Annualized Value of Risk-Free Individual Benefits, $r = 4.0\%$

Estimated using current CalSTRS assumptions and benefit formula for new hires, slightly modified.

The curves depict $n_{es}$, the annualized value, as a percent of earnings, for risk-free benefits of an individual entering at age $e$ and exiting at age $s$. Variation by age of exit is shown along each curve; variation by age of entry is shown across curves.

The curves depict $n_{es}$, the annualized value, as a percent of earnings, for risk-free benefits of an individual entering at age $e$ and exiting at age $s$. Variation by age of exit is shown along each curve; variation by age of entry is shown across curves.